Effects of Increasing Enforcement on Firm Value and Financial Reporting Quality

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Abstract

A standard assumption in empirical research and capital markets policy making is that increasing enforcement effectiveness improves financial reporting quality. In this paper, we show that this relation does not generally hold, even if enforcement is costless. We develop an agency model with a productive manager who can also engage in earnings management, a strategic auditor, and an enforcement institution. We establish the equilibrium strategies and the optimal management compensation. Our main result is that firm value and financial reporting quality can decrease, typically if enforcement becomes too strong. One reason is that enforcement and auditing are complements under weak enforcement, but are substitutes under strong enforcement. Less auditing reduces reporting quality. The other reason is that earnings management can be "good" if it corrects errors by an imprecise accounting system; mitigating earnings management reduces this corrective effect, which also lowers quality.

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1. Introduction

Enforcement assists in assuring the quality of financial reporting by listed companies through supervision of published audited financial reports. Many countries have established enforcement institutions, such as the SEC's Division of Corporation Finance in the U.S. and national enforcement agencies in EU countries that are overseen by the European Securities and Markets Authority (ESMA). Effective enforcement has been identified in many studies as being crucial for the efficiency of capital markets and perhaps more important than the quality of the accounting standards themselves (e.g., Ball, Kothari, and Robin 2000; Christensen, Hail, and Leuz 2013). Currently, the effectiveness of enforcement institutions differs widely around the world (Brown, Preiato, and Tarca 2014), and regulators strive to improve enforcement to foster capital market efficiency (e.g., SEC 2000, EU 2004).

A maintained assumption in empirical research and policy making in capital markets is that increasing enforcement is desirable because it improves financial reporting quality, and several empirical studies provide evidence that is consistent with this assumption.[1] Under this view it is solely the direct cost of enforcement that prohibits full enforcement. This paper rigorously examines this assumption and shows that increasing enforcement, even if it is costless, can be detrimental for firm value (welfare) and for financial reporting quality. Intuitively, there are two reasons why more enforcement can be undesirable: First, enforcement focuses on compliance and, thus, is narrower in scope than auditing that also takes into account fair presentation; we show that too effective enforcement crowds out auditing, which lowers reporting quality. Second, earnings management can be "good" if it corrects random errors in the accounting process; because enforcement reduces earnings management, it also reduces its positive correction effect.

To establish our results, we develop an agency model with a manager who exerts productive effort and can engage in earnings management, a strategic auditor, and an

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[1] See, e.g., Hope (2003), Ernstberger, Stich, and Vogler (2012), Christensen, Hail, and Leuz (2013), Brown, Preiato, and Tarca (2014).

enforcement institution. The optimal contract that induces the manager to exert productive effort also creates incentives for earnings management. The auditor strategically chooses the audit effort based on his conjecture of earnings management and corrects errors found in the preliminary financial report. A key driver of our results is that auditing and enforcement are different activities. Auditing comprises the quality of the accounting system and internal controls as well as earnings management, whereas the scope of enforcement is more limited and geared towards detecting earnings management.

After publication of the audited financial report, the enforcer supervises the report and identifies further errors. If the auditor is unable to provide evidence that the alleged error is in fact nonexistent, the enforcer takes an enforcement action, which imposes enforcement to the firm, to the auditor, and through claw-back of a bonus also to the manager. We derive equilibrium earnings management and audit effort and the optimal compensation contract, and we study the economic effects of a change in enforcement effectiveness on the equilibrium.

Our main findings are the following: First, we confirm the result that equilibrium earnings management strictly decreases with stronger enforcement. Second, we show that firm value is always higher for perfect enforcement than no enforcement at all, but varying existing enforcement can either increase or decrease firm value, contingent on key parameters of the economic situation. In particular, we show that generally an imperfect enforcement level is optimal. Third, increasing enforcement can either improve or reduce financial reporting quality, and we provide necessary conditions in which one or the other happens. Counterintuitively, financial reporting quality can strictly decrease for an increase in enforcement. Fourth, we find that financial reporting quality and firm value can move in parallel, but also in different directions; thus, increasing enforcement may improve financial reporting quality, but destroy firm value, and vice versa. Finally, we discuss empirical implications of our analyses.

Two reasons are jointly responsible for why increasing enforcement can have negative effects on firm value and financial reporting quality. One reason is that increasing enforcement from a low level raises incentives of the auditor to increase audit effort because enforcement actions are costly to all players, including the auditor. Both effects mitigate earnings management and correct accounting errors. However, if enforcement becomes sufficiently strong, enforcement becomes more effective in deterring earnings management, and in equilibrium the auditor reduces audit effort. That is, whereas auditing and enforcement are complements for weak enforcement, they become substitutes for strong enforcement. Because auditing is broader in scope than enforcement, a decrease in audit effort reduces the quality of the financial reporting system.

The other reason is that earnings management is not necessarily "bad" in that it obscures information. The optimal contract provides incentives to the manager to overstate earnings. This overstatement is "bad" if actual earnings are low because it disguises this fact, but it is "good" if it corrects an erroneous financial report that shows low earnings, although the actual outcome is high. The latter effect becomes more likely if the accounting system is less precise and we give a condition earnings management is "good" on average. Because more effective enforcement unambiguously reduces earnings management, it also reduces "good" earnings management, which is undesirable.

This paper contributes to the accounting and auditing literature by examining the economic effects of enforcement on the main two objectives of financial reporting, decision usefulness and stewardship, directly and indirectly through auditing in equilibrium. We are not aware of other analytical papers that explicitly study economic effects of enforcement and particularly its interaction with auditing.

The productive setting in the present paper is related to work that studies production effort and earnings management in multi-action agency models. For example, Feltham and Xie (1994) model productive effort and earnings management ("window dressing"), which are simultaneously induced by the same information system, and provide insights into the properties of an optimal information system in a LEN setting. Glover and Levine (2015) consider asymmetric information about measurement quality and show that earnings management can be "good" in that it reduces understatement; a similar feature emerges in our paper. Laux and Laux (2009) study management compensation by the board of directors, who also decide on their oversight effort, and show that these two decisions are related. Bertomeu, Darrough, and Xue (2015) consider production and earnings management choices and focus on the optimal bias (conservatism) of the underlying accounting system. Laux and Stocken (2015) study a similar setting, but focus on the interaction between accounting standards and enforcement. Enforcement in their model discovers non-compliance with some probability and imposes a penalty that increases with stronger enforcement. Neither of these papers considers auditing and enforcement jointly.

Other models study earnings management in rational expectations equilibria, in which managers “jam” financial reports to increase the market price of the firm (see, e.g., Fischer and Verrecchia (2000); Ewert and Wagenhofer 2011 survey this literature). In these models, auditing and enforcement are implicit in the cost of earnings management. Königsgruber (2012) addresses enforcement in a model in which a manager decides on the investment in a risky project and is concerned about the market price of the firm after issuing a financial report. Enforcement in his paper is a technology that reveals the true outcome with a probability that is set *ex ante* by a regulator and imposes a fine after detecting misreporting. Königsgruber finds that more effective enforcement strictly increases reporting quality, but may reduce investment efficiency due to over-deterrence of viable projects.[2] Different from that, our results show that both reporting quality and investment can decrease; the reason is that we explicitly model the interaction between auditing and enforcement.

The auditing literature analyzes audit strategies, but does not explicitly introduce enforcement. Some papers assume a strategic auditor, who maximizes expected utility by the choice of audit effort (Antle 1982, Baiman, Evans, and Noel 1987), as we do in the present paper. Given that contingent audit fees are not allowed in most jurisdictions, the motivation for auditors to exert audit effort in these models usually results from the risk that the auditor is held liable of malperformance if an error in the financial reports is uncovered later. The enforcement mechanism in the present paper is explicitly modeled based on its interaction

[2] Deng, Melumad, and Shibano (2012) find a related result for increased auditor liability.

with the audit results. Other papers assume that the liability arises from shareholder litigation. In that case, the cost to the auditor depends on decisions taken in a rational fashion by shareholders and on the liability regime (e.g., Ewert 1999, Hillegeist 1999). Related to the present paper is the audit literature that also considers internal controls, if one views internal controls as an assurance mechanism that steps in before auditing takes place (e.g., Smith, Tiras, and Vichitlekarn 2000, Pae and Yoo 2001). In the present model, we explicitly model enforcement and study its interaction with auditing effort.

The paper proceeds as follows. In Section 2, we set out the model and introduce the underlying production technology, the accounting system, the discretion for earnings management, auditing, and enforcement. Section 3 contains the analysis of the earnings management and auditing game, which depends on the enforcement. Section 4 adds the production stage and derives the optimal compensation contract with the manager, which generates the incentives for earnings management that affect the subsequent reporting equilibrium. We show how enforcement affects the owner's expected utility, which is equivalent to firm value in our setting. In Section 5, we extend our analysis to the consequences of varying enforcement on financial reporting quality. Section 6 contains robustness checks, and Section 7 concludes and summarizes empirical implications.

2. Model

We develop a one-period agency model with a representative owner of a firm, a manager, an auditor, and an enforcement institution (the "enforcer"). In the following, we describe these elements and their relation step by step. The notation is summarized in the appendix.

Production technology

The owners of the firm are represented by a risk neutral owner (or the board of directors to which the decision power is delegated). We abstract from potential conflicts of interest among different owners or among owners and board members. The firm owns a production technology and has an accounting system in place. The production technology requires the input of a manager (effort a), which, together with random events capturing other productive and environmental factors, determines the outcome. The output is represented by a monetary amount x, where $x \in \{x_L, x_H\}$ and $0 < x_L < x_H$. We adopt the convention that x denotes the random variable and x_i ($i = L, H$) its discrete realizations. The owner receives the output of the production technology and pays the compensation to the manager $s(\cdot)$.

The owner hires a manager, who is risk neutral and protected by limited liability. The manager chooses a productive effort $a \in \{a_L, a_H\}$ and incurs a private cost of 0 for a_L and $V > 0$ for a_H. The effort determines the probability with which a low and a high output realize: x_H occurs with probability p upon high effort a_H, and with probability q upon low effort, where $p > q$ and each p and q are strictly within (0, 1).

We focus on the case that the owner wants to induce the manager to exert high productive effort a_H, because otherwise there is no agency problem. We assume that x is unobservable throughout the time period we examine; for example, the output can be the expected net present value of future cash flows.[3] The firm operates an accounting system and issues an audited financial report r. This report is contractible and is used in the manager's compensation contract to elicit managerial effort.

The owner maximizes the expected utility that includes the following components: the expected productive outcome $(1 - p)x_L + px_H$ less expected compensation $\text{prob}(r_L)s(r_L) + \text{prob}(r_H)s(r_H)$, the audit fee A, and the expected costs due to an enforcement action.

Accounting system

The firm operates an accounting system that produces a signal $y \in \{y_L, y_H\}$, where $y_L < y_H$ (see Figure 1). We also refer to these signals as earnings. The accounting system is an imperfect "technology" subject to possible random errors and accounting standards that may

[3] This assumption precludes writing a contract contingent on x. A qualitatively similar assumption is that the owner may sell the shares after the financial report has been issued and the manager was paid. To price the shares, capital market participants use the report about the future cash flow x.

produce biases. α is the "α-error", i.e., the probability that y_L is reported although the output is x_H; and β is the "β-error" with which y_H is reported although the output is x_L. α and $\beta \in (0, \frac{1}{2})$ are exogenously determined by the accounting standards and their implementation in the firm and are common knowledge. The manager privately observes the accounting signal y; hence, y is not available for contracting.

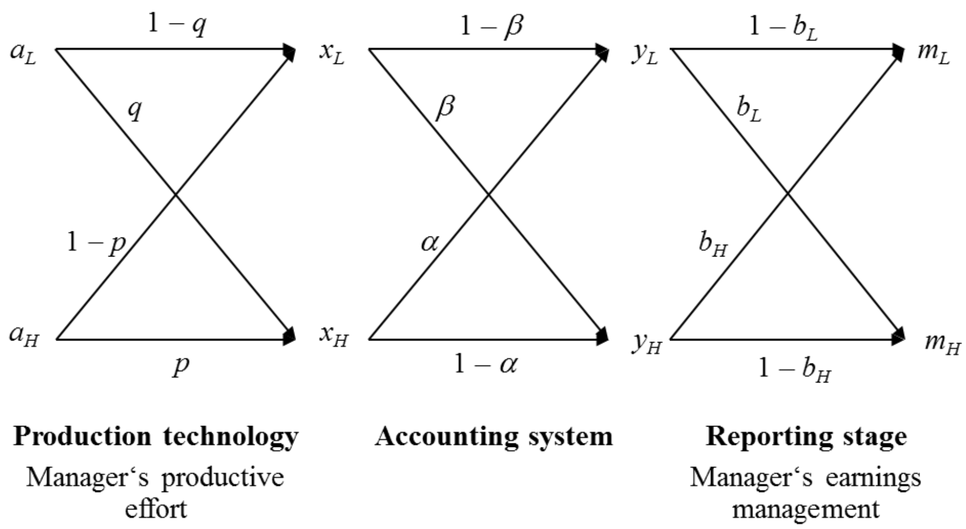

Figure 1: Production and reporting structure

After observing y, the manager can engage in earnings management and misrepresent the signal to achieve a financial report $m \neq y$. We refer to the report m as the preliminary financial report because it is subject to auditing (see below). Earnings management includes the choice of probabilities $b_L \equiv b(y_L)$ and $b_H \equiv b(y_H)$ with which it is successful in diverting the accounting signal, i.e., reporting $m_i \neq y_i$, $i = L, H$. The cost of earnings management effort is increasing and convex in b_i, it is 0 at $b_i = 0$, and "very high" at $b_i = 1$. It captures disutility from, e.g., searching for earnings management opportunities, future disadvantages, reputation, or ethical behavior. For tractability reasons, we assume a quadratic cost function, $\frac{1}{2} v b_i^2$, where v is a constant scaling factor. We assume that v is sufficiently high that $b_i < 1$ (such a

v always exists)[4] in order to avoid consideration of cases in which $b_i = 1$ and the financial report becomes uninformative.

The manager receives compensation from the owner for the effort. We assume the manager has a reservation utility of zero and because of limited liability the compensation paid must be positive. Compensation $s(\cdot) \geq 0$ is written on the audited financial report $r \in \{r_L, r_H\}$, which is the contractible signal. Finally, the audited report is subject to enforcement. If the enforcer finds and publishes an error, we assume the owner invokes a claw-back of a bonus paid to the manager, thus penalizing the manager for identified misrepresentation. The claw-back imposes a contingent element in the otherwise simple bonus contract. We do not consider more complex compensation contracts.

Auditing

The firm is subject to mandatory auditing. The owner contracts with an auditor prior to the preparation of the preliminary report m by the manager. The audit comprises tests of controls and substantive procedures, including analytical procedures and tests of details, e.g., providing audit evidence of physical inventory, bank balances, loan quality, and the like, to identify material misstatements. After engagement, but before deciding on audit effort, the auditor receives the preliminary report m from the manager, but no other information. The auditor knows the precision of the accounting system (α, β) and uses it for risk assessment. Performing the audit, the auditor observes both the actual accounting signal y and the true outcome x with a probability that increases in audit effort. For example, the auditor may have proprietary industry expertise. Let g_i be the probability with which the auditor finds out (x, y) given m_i, $i = L, H$. Providing audit effort g_i is privately costly to the auditor; the cost is $\frac{1}{2}kg_i^2$, where $k > 0$ is a parameter that scales the quadratic cost.

The actual outcome x is always more informative about the firm's cash flows than the accounting signal y, and therefore we assume the auditor corrects the financial report based on

[4] In the proof of Proposition 2, we derive the precise condition as $v > 2V / [(p - q)(1 - \alpha - \beta)]$.

x.[5] That is, if the auditor finds out that m_i has been reported but the outcome is x_j, $i \neq j$, ($i = L$, H) then he requires the manager to correct the financial report from m_i to r_j;[6] if $m_i = x_i$, no action is required and $r_i = m_i$. The audited financial report is as follows:

$$r_i = \begin{cases} x_i & \text{with probability } g_i \\ m_i & \text{with probability } (1 - g_i) \end{cases} \tag{1}$$

The probabilities that the auditor finds and corrects an error, conditional on m_i, are $\text{prob}(x_H | m_L) g_L$ and $\text{prob}(x_L | m_H) g_H$. Note that r is more informative in the terms of fineness than m with respect to x because r is a combination of m and x. In the extreme case, a perfect audit ($g_i \to 1$) always reveals x, making m useless; we rule out this case by assuming k is sufficiently large to ensure that $g_i < 1$ for $i = L, H$.

The audit market comprises auditors with similar characteristics and is competitive. Capturing the requirements of typical audit regulations, we assume that the audit fee $A > 0$ is constant (and not contingent on the auditor's report) and determined by negotiation between the owners of the firm and the auditor. Under the assumed market conditions, A is the fee with which the auditor expects to break even on his engagement. After accepting the engagement, the auditor's objective is the minimization of the expected cost of the audit and of costs resulting from any remaining uncorrected errors that are identified by enforcement. In case of an enforcement action, the auditor incurs a cost $C^A > 0$. Assuming $C^A/k \leq 1$ is sufficient to ensure $g_i < 1$.[7]

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[5] Given our assumptions, the auditor would be indifferent between correcting m to x or y because the enforcer only observes y (as we discuss below) and the auditor can provide evidence that the actual outcome is indeed x. We rule out other correction strategies by assuming that the auditor cares for higher-quality reports if indifferent and discuss this assumption in the Discussion and Conclusions section.

[6] We assume that if the manager does not correct the report the auditor issues a qualified audit opinion, which has the same informative effect.

[7] Note that this assumption does not imply that the amount of the penalty is lower than the cost of effort. The effort cost depends on g_i, which is 0 at $g_i = 0$, but increases to a large amount if $g_i \to 1$. In equilibrium, we show later that C^A is greater than the effort cost $k g_H^{2*} / 2$.

Enforcement

Enforcement is an institution that independently investigates published audited financial reports. The scope of enforcement is limited and the enforcer does not perform another audit. While the audit includes both tests of controls and substantive procedures, enforcement performs limited investigations that often include few positions that are considered critical and particularly focuses on compliance with accounting standards. In many environments, the enforcer even preannounces accounting issues that it focuses on, such as impairments, consolidation, deferred tax assets, and the like, which require significant judgment by management and are prone to earnings management. To model the difference between enforcement and auditing parsimoniously, we assume the investigation by the enforcer, after observing the audited report r_i, uncovers the signal y_j from the accounting system with some probability f (referred to as enforcement effectiveness) , but not the actual outcome x. As a consequence, auditing is always more comprehensive than enforcement and provides more information per unit of effort. However, the activities uncover different errors because the auditor's and the enforcer's probabilities of detecting errors are uncorrelated.

The enforcer operates on a fixed budget, which we assume as exogenously determined by a governmental institution.[8] In our model, the budget determines the probability $f \in [0, 1]$ with which the enforcer detects y. A higher budget increases f. Without loss of generality, we cast our analysis in terms of f directly.

If the enforcer obtains y_i, a report r_i that equals y_i ($i = L, H$) ends the investigation without a finding. If the report r_i deviates from y_j, $i \neq j$, then the enforcer alleges an error has occurred. If the firm or the auditor can present evidence that $r_i = x_i$, the enforcer accepts this and ends the investigation. However, if no such evidence is available, the enforcer declares an error in the financial report, which is published, and subjects the parties involved to penalties.

[8] We do not consider the possibility that firms directly or indirectly pay for the enforcement to isolate the strategic effects from direct cost effects. Taking direct costs of enforcement into account would reinforce our main result that more enforcement can be detrimental.

We assume that presenting evidence is costless to the auditor because he already collected it during the audit, and there is no further search for evidence in case the enforcer alleged an error.

The firm's costs of an enforcement action are a potential loss of reputation and credibility of its financial reports, penalties, and other costs of legal liability. We denote these costs by $C^O > 0$. We do not explicitly model shareholder litigation against the firm, the manager, or the auditor.[9] The manager is protected by limited liability, and we assume there are no other costs, such as a loss of reputation, or personal sanctions imposed. Therefore, the sole consequence of an enforcement action is a claw-back of compensation paid from an erroneous report *r*, which is paid back to the firm's owners. Finally, the costs to the auditor C^A include penalties, fines, potential legal liability, but also indirect effects such as a reputation loss.

Figure 2 summarizes the sequence of events. The subsequent analysis is by backward induction: We begin with analyzing the effectiveness of enforcement and then turn to the reporting equilibrium that consists of the auditor's decision problem and the manager's earnings management decision. Next, we examine the productive effects of enforcement by analyzing the manager's productive effort choice. Using the results, we then examine the owner's problem of designing the manager's compensation contract and determine the effects of enforcement on the owner's expected utility. In the last step, we consider the effects of increasing enforcement on equilibrium financial reporting quality. All proofs are in the appendix.

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[9] Litigation requires that there exists a mechanism that *x* becomes eventually observable. We believe that the introduction of a litigation stage does not materially affect our main results.

- Owner offers contract to manager and engages auditor
- Manager provides productive effort *a*
- Manager observes accounting signal *y* and engages in earnings management *b*
- Preliminary report *m* is realized
- Auditor chooses audit effort *g*, learns (*x*, *y*) and corrects errors ($m \neq x$) in the preliminary report
- Audited report *r* is publicly issued
- Manager receives contractual compensation *s*(*r*)
- Enforcer investigates audited report *r*, learns *y* and alleges error ($r \neq y$)
- Auditor may provide evidence that no error occurred ($r = x$ although $r \neq y$); otherwise publication of error and enforcement action
- Firm, manager, and auditor incur costs from enforcement action

Figure 2: Time line

3. Reporting equilibrium

3.1. Preliminary results

We start with a preliminary result on the structure of the compensation function and the manager's earnings management decision, which simplifies the rest of the analysis.

The manager's expected utility, given the high productive effort a_H, is[10]

$$E[U^M | a_H] = \underbrace{\text{prob}(r_L)s(r_L) + \text{prob}(r_H)s(r_H)}_{\text{Expected compensation}} - \underbrace{V}_{\text{Cost of productive effort}}$$
$$- \underbrace{\frac{v}{2}\left(\text{prob}(y_L)b_L^2 + \text{prob}(y_H)b_H^2\right)}_{\text{Cost of earnings management}} - (\text{expected cost of claw-back}) \quad (2)$$

The owner wants to induce the manager to exert effort a_H through the contractual compensation *s*(*r*) promised to the manager.

[10] Note that the probabilities are contingent on a_i. To save notation, we do not explicitly write this dependence if $a = a_H$.

***Lemma* 1**: The optimal contract to induce a_H is characterized by $s(r_H) > s(r_L) = 0$. Furthermore, $b_H = 0$.

This result is intuitive: First, to induce the manager to exert high effort at a personal cost V, the compensation must be greater for the report that is more likely with a_H than with a_L, which is r_H because $\text{prob}(r_H \mid a_H) > \text{prob}(r_H \mid a_L)$. Therefore, $s(r_H) > s(r_L)$. Second, there is no reason to pay the manager more than his reservation utility, therefore, $s(r_L) = 0$, the minimum payment in this case. We label $s \equiv s(r_H)$ the bonus. Given this compensation structure, the manager has an incentive to engage in earnings management if she observes y_L to increase the probability of a report m_H, but no incentive for earnings management if she observes y_H, which is $b_H = 0$.

3.2. Enforcement action

The enforcement affects all decisions taken prior to it because the parties consider the subsequent effects in their decisions. The two panels in Figure 3 depict the events evolving after the manager observes the accounting signal y_L and y_H, respectively, and the conditional probabilities of the events.

The first panel in Figure 3 depicts the events if $y = y_L$ is realized. In this case, the manager engages in earnings management $b_L \geq 0$. If it is unsuccessful (probability $1 - b_L$), the preliminary report remains m_L. The auditor finds out x with probability g_L: if $x = x_H$, the auditor requests that the preliminary report be corrected to r_H; otherwise, the audited report is r_L and if enforcement does not unravel y, no error is detected.[11] If the enforcer learns y, then it is y_L, hence again there is no error. If the audited report is r_H, it is not challenged if the enforcer does not learn y. If it finds out y (probability f), it is $y = y_L$, and the enforcer alleges an error because $r_H \neq y_L$. However, this case can only occur under $y = y_L$ if the auditor corrected the preliminary report based on his observation of x_H; therefore, he will provide evidence to the enforcer that there is in fact no error.

[11] Lemma 2 below establishes $g_L = 0$.

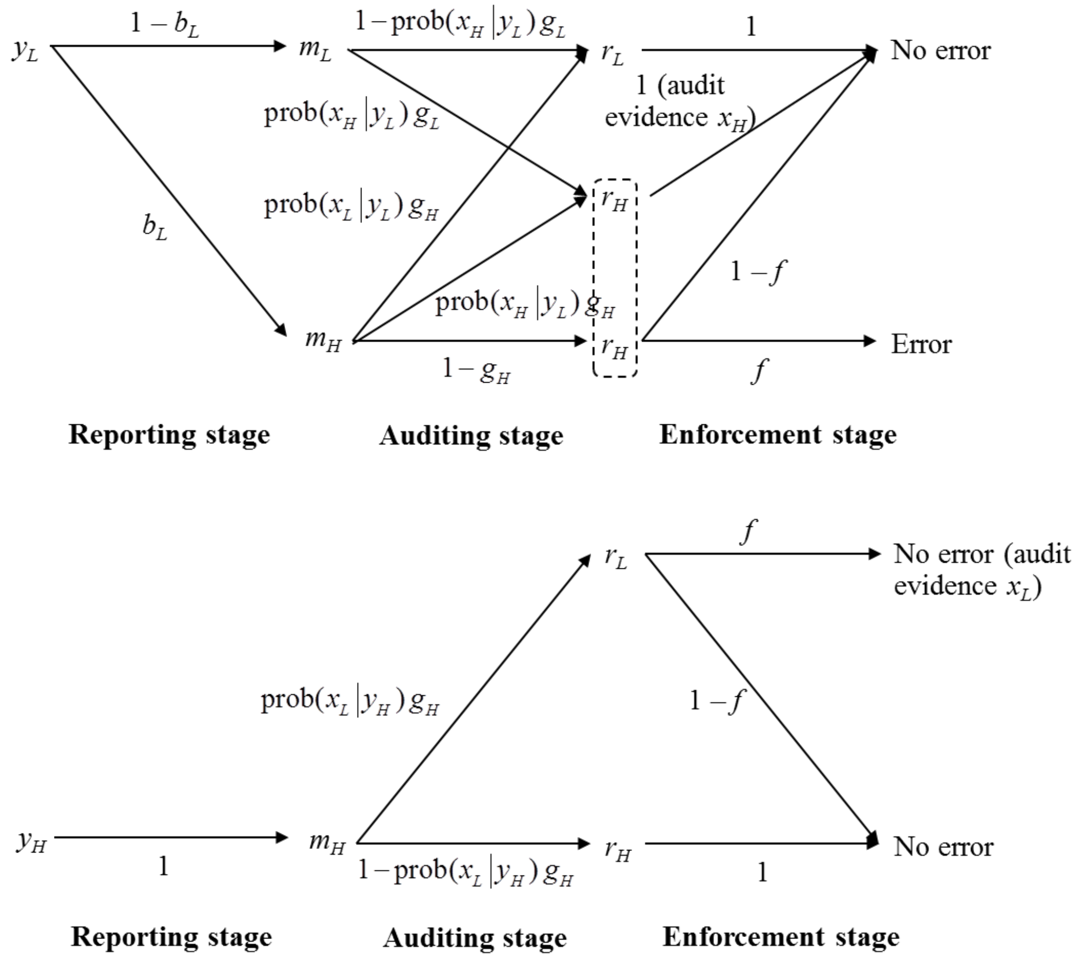

Figure 3: Auditing and enforcement stages

If y_L is realized and earnings management is successful (probability b_L), the preliminary report is m_H. Again, if the auditor learns x, he will request correction to r_L ($\text{prob}(x_L|y_L)\,g_H$). Because $r_L = y_L$, regardless of whether it observes y or not, the enforcer will not find an error. If the auditor learns $x = x_H$, no correction is made because the enforcer finds out $y = y_L$ with probability f, but there is evidence that $r_H = x_H$ is correct. Finally, if the auditor did not find out y (probability $1 – g_H$) and the enforcer finds out $y = y_L$, it alleges an error, which the auditor cannot object, and this is the only case in which an error is published and an enforcement action is triggered.

The second panel in Figure 3 shows the events for $y = y_H$. Because there is no earnings management ($b_H = 0$ by Lemma 1), the only situation in which $r = r_L$ results from the auditor learning x and observing $x = x_L$, which occurs with $\text{prob}(x_L | y_H) g_H$. In this case the auditor requests correction, and the audited report is r_L. If the enforcer does not learn y it cannot find an error; if it learns y, it will allege an error because $y_L \neq r_H$. However, in this case the auditor will present evidence that the report $r_H = x_H$ is correct. That is, if y_H is realized, enforcement never finds an error.

Taken together, an error found by enforcement can only occur in one particular constellation: the accounting system reports low earnings, the manager succeeds in managing earnings upwards, the audit does not uncover this bias, and the enforcer observes the low accounting signal. Note, however, that even in this case, the resulting financial report is not free of error, because the enforcer does not observe the outcome x that is ultimately relevant.

3.3. Audit effort

Given the auditor accepted the audit engagement, he determines the audit effort g_i by maximizing the expected utility conditional on the preliminary report m_i,

$$U^A(m_i) = A - \frac{k}{2} g_i^2 - \text{prob}(\text{error} | m_i) C^A \tag{3}$$

where A is a constant at this stage.

***Lemma* 2**: The optimal audit effort levels are:

$$g_L = 0 \text{ and } g_H = \text{prob}(y_L | m_H) f C^A / k$$

where $g_H > 0$ if $\hat{b}_L > 0$ and $f > 0$.

The incentive of the auditor to provide audit effort results from the risk of an enforcement action, the cost of which is captured by the last term in his utility function (3), $\text{prob}(\text{error} | m_i) C^A$. Higher audit effort increases effort cost, but reduces the probability of an enforcement action that is costly.

As is apparent from Figure 3, there is no risk of an enforcement action if the preliminary report is m_L, because this case can only occur if accounting earnings are y_L and the manager's earnings management was unsuccessful (the manager never engages in earnings management if y_H obtains because $b_H = 0$). Therefore, the auditor optimally chooses $g_L = 0$. In contrast, if the preliminary report is m_H, the auditor has an incentive to exert audit effort $g_H > 0$. The reason is that he faces the risk that the enforcer finds an (undisputed) error, that is, $\text{prob}(\text{error}|m_H) > 0$ if he conjectures that the manager engaged in earnings management $(\hat{b}_L > 0)$ and if enforcement exists ($f > 0$). The error probability given m_H is

$$\text{prob}(y_L|m_H) = \frac{\text{prob}(y_L)\hat{b}_L}{\text{prob}(y_L)\hat{b}_L + \text{prob}(y_H)}$$

which is 0 for $\hat{b}_L = 0$ and increases in $\hat{b}_L$; therefore, g_H increases in $\hat{b}_L$ as well. The audit effort also depends on the probability f that the enforcer finds out y. If $f = 0$, the auditor anticipates that there is no enforcement and has no incentive to provide audit effort. For $f > 0$, audit effort increases in f. Finally, the term C^A/k captures the relative cost of an enforcement action and audit effort.

Given the optimal audit effort, the auditor's conditional utility equals

$$\begin{aligned} U^A(m_H) &= A - \frac{k}{2} g_H^2 - \text{prob}(y_L|m_H)(1-g_H) fC^A \\ &= A - \frac{k}{2} g_H(2-g_H) \end{aligned}$$

The auditor accepts the audit engagement if the expected utility is greater or equal to zero. In a competitive audit market with homogenous auditors the expected profit of the auditors is zero. If $m = m_H$, A must at least equal $A = kg_H(2-g_H)/2$; if $m = m_L$, the auditor exerts no effort and $A = 0$. Therefore, *ex ante* the audit fee is

$$A = \text{prob}(m_H)\left(\frac{k}{2} g_H(2-g_H)\right). \tag{4}$$

Note that A depends on the conjectured earnings management strategy $\hat{b}_L$ directly through g_H and indirectly through $\text{prob}(m_H)$.

3.4. Earnings management effort

The manager makes the earnings management decision based on the realized accounting signal y that she privately observes. In Lemma 1 we establish that $s(r_H) = s > 0$, $s(r_L) = 0$, and $b_H = 0$, that is, the manager never misreports after observing y_H. In Lemma 2 we show that $g_L = 0$ and g_H increases in the auditor's conjecture of earnings management $\hat{b}_L$. To determine b_L, the manager maximizes her expected utility conditional on y_L and the conjecture of the audit effort $\hat{g}_H$:

$$E[U^M|a_H, y_L] = \text{prob}(r_H|y_L)s - V - \frac{1}{2}vb_L^2 - b_L(1-\hat{g}_H)fs \tag{5}$$

where the last term, $b_L(1-\hat{g}_H)fs$, captures the cost of enforcement to the manager, which equals the probability that the enforcer finds an error given y_L multiplied by the bonus s that must be paid back.

The benefit of earnings management is that b_L increases the probability that the preliminary report is m_H if the accounting signal is y_L, which increases the probability of receiving a bonus , which is

$$\text{prob}(r_H|y_L) = b_L(1-\hat{g}_H) + b_L\,\text{prob}(x_H|y_L)\hat{g}_H + \underbrace{(1-b_L)\,\text{prob}(x_H|y_L)\hat{g}_L}_{=0}$$

Lemma **3**: Given some s, earnings management decreases in the conjectured audit effort $(\partial b_L/\partial\hat{g}_H < 0)$ if and only if

$$T \equiv \text{prob}\left(x_H|y_L\right) - \left(1-f\right) < 0 \tag{6}$$

The lemma follows directly from the first-order condition of $E[U^M|a_H, y_L]$ with respect to b_L,

$$\begin{aligned} b_L &= \frac{s}{v}\left((1-\hat{g}_H)(1-f) + \hat{g}_H\,\text{prob}(x_H|y_L)\right) \\ &= \frac{s}{v}[(1-f) + \hat{g}_H\underbrace{\left(\text{prob}(x_H|y_L) - (1-f)\right)}_{\equiv T}] \end{aligned}$$

Intuitively, one would expect that misrepresentation always decreases if the conjectured audit effort $\hat{g}_H$ increases. However, this relation holds only if the term $T \equiv \text{prob}(x_H|y_L) - (1-f) < 0$. Ceteris paribus, misrepresentation decreases in audit effort only if enforcement f is “low”; whereas it *increases* in f if f is “high”. To see why, note that a higher $\hat{g}_H$ increases the probability that the auditor finds out the true x, which has two opposing effects: (i) it reduces the probability of receiving a bonus because the auditor detects x, including x_L, more often and a bonus requires that the auditor does not find out x and enforcement is unsuccessful, which occurs with probability (1 – f). (ii) However, if the auditor finds out x, it can also be x_H, which promises the manager a bonus regardless of enforcement. The probability of this second effect is

$$\text{prob}(x_H \,|\, y_L) = \frac{p\alpha}{p\alpha + (1-p)(1-\beta)}$$

That is, the manager implicitly increases earnings management to induce more auditing, which is beneficial in this case. The optimal b_L trades off these two effects, and this trade-off is captured in T. An increase of b_L in $\hat{g}_H$ is more likely if the enforcement level f is relatively high and/or the accounting system is less precise (i.e., α is relatively high).

The next result establishes a unique equilibrium in this manager-auditor game, which includes both earnings management and audit effort.

***Proposition* 1**: Given some s that induces a_H and $f \in (0, 1)$, there exists a unique equilibrium with earnings management $b_L^* > 0$ and audit effort $g_H^* > 0$.

The equilibrium earnings management b_L^* and audit effort g_H^* depend in a complex way on all relevant parameters. The proof in the Appendix gives explicit expressions for b_L^* and g_H^*. In the following subsection, we provide comparative statics results.

3.5. Effects of enforcement on the reporting equilibrium

We examine the effects of enforcement effectiveness f and the costs of enforcement actions C^A. We also consider the effects of variations in the bonus payment s; we endogenize s in the subsequent section. Note that the owner's cost of enforcement C^O has no effect on the reporting equilibrium because it affects neither the manager nor the auditor. Its only effect is that it raises the cost of motivating high productive effort a_H, which ultimately may lead the owner to prefer the low effort a_L.

***Corollary* 1**: Assume some s that induces a_H. Equilibrium earnings management and equilibrium audit effort have the following properties:

(i) b_L^* strictly increases in s for $b_L^* > 0$, and g_H^* strictly increases in s for $g_H^* > 0$;

(ii) b_L^* strictly decreases in f, and g_H^* strictly increases in f for $f < f_0$ and strictly decreases for

$f > f_0$, where $1/2 < f_0 < 1$;

(iii) b_L^* strictly decreases in C^A/k if and only if $T < 0$, and g_H^* strictly increases in C^A/k.

Corollary 1 (i) establishes that both b_L^* and g_H^* strictly increase in the bonus payment. A greater s increases *ceteris paribus* the marginal benefit of earnings management, which provides stronger incentives to the manager to work hard and to engage in earnings management. A higher conjecture of earnings management induces higher audit effort. However, the higher audit effort mitigates earnings management, which works against the direct increase through higher s. Corollary 1 (i) shows that in equilibrium the net effect is still an increase in earnings management.

Corollary 1 (ii) confirms the intuitive result that earnings management strictly decreases in enforcement effectiveness f. If enforcement becomes perfect ($f \rightarrow 1$), it eliminates earnings management altogether. In contrast, the effect of a change in the enforcement effectiveness on the equilibrium audit effort depends on the level of enforcement: Starting from $f = 0$, increasing f increases g_H^*, which results from the increase in the expected cost of enforcement to the auditor. However, there is an enforcement level $f_0 > 1/2$ at which g_H^* achieves its maximum and increasing enforcement further reduces g_H^*, until it approaches 0 for $f \rightarrow 1$, because perfect enforcement eliminates earnings management, which again takes away any enforcement risk and any audit incentives from the auditor. This result suggests a complementary relation between audit effectiveness and enforcement effectiveness if enforcement is weak, and a substitutive relation between the two if enforcement is strong.

Corollary 1 (iii) states the effect of a variation of the cost of an enforcement action C^A to the auditor and a variation of the audit effort cost parameter k. The important parameter is the ratio C^A/k, which captures the relative enforcement cost over the scaling parameter k on audit effort cost. The enforcement cost provides the incentive for the auditor to exert effort; a direct consequence of this is that audit effort increases in C^A (decreases in k). Given higher audit effort, one would expect a reduction of equilibrium earnings management. However, Corollary 1 (iii) states this holds only if

$$T = \frac{1}{1+\frac{(1-p)(1-\beta)}{p\alpha}} - (1-f) < 0 .$$

Otherwise, b_L^* strictly increases in C^A (decreases in k). Recall that Lemma 3 establishes that $\partial b_L / \partial \hat{g}_H > 0$ if $T > 0$ and vice versa,[12] and the reason for the result in Corollary 1 is similar. The manager's optimal bias given y_L is

$$b_L^* = \frac{s}{v}\left((1-g_H^*)(1-f) + g_H^* \operatorname{prob}(x_H \mid y_L)\right)$$

A greater C^A (lower k) increases the audit effort, and this has two effects on the bias: (i) higher audit effort increases the probability that the auditor detects the true outcome, which is beneficial for the manager if the auditor finds x_H because the manager receives the bonus without a risk of a claw-back in case of effective enforcement. (ii) Higher audit effort reduces the probability of a bonus if the auditor is unsuccessful in identifying the true outcome. Here a claw-back can arise after enforcement, thus only the net loss of the bonus is relevant. The term T captures the trade-off between these two effects: If T is positive, the positive effect dominates, thus leading to higher earnings management; and vice versa.

4. Optimal compensation contract

4.1. Owner's decision problem

We now turn to the first stage in the game, in which the owner hires the manager and offers a compensation contract that induces the manager to exert high effort a_H. Our preliminary results in Lemma 1 record basic properties of the optimal contract: it is a bonus contract with $s(r_H) = s > 0$ and $s(r_L) = 0$. In determining the optimal compensation, the owner must consider that a higher bonus s increases the manager's incentive to work hard, but also increases her incentive to engage in earnings management. Recall that Corollary 1 (i) establishes that equilibrium earnings management strictly increases in s, which again affects the equilibrium audit effort and the cost of enforcement.

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[12] It is noteworthy that the equilibrium strategies behave differently to the more intuitive behavior of the reaction functions.

The owner maximizes the expected utility with regard to s, taking into account the subsequent equilibrium strategies it triggers. The expected utility comprises the following components:

$$E[U^O | a_H] = \underbrace{(1-p)x_L + px_H}_{\text{Expected outcome}} - \underbrace{\text{prob}(r_H)s}_{\text{Expected compensation}} - \underset{\text{Audit fee}}{A} - \underbrace{\text{prob(error)}C^O}_{\substack{\text{Expected cost}\\ \text{of enforcement}}} + \underbrace{\text{prob(error)}s}_{\substack{\text{Expected claw-}\\ \text{back of bonus}}} \quad (7)$$

Because the expected outcome depends only on the production technology, the owner minimizes the expected compensation to the manager with respect to the bonus s, considering the (endogenous) audit fee and the net cost of an error identified through enforcement. An enforcement action costs the firm C^O, net of a claw-back of the manager's bonus. The owner's objective function becomes

$$\min_s \left(\text{prob}(r_H)s + A + \text{prob(error)}(C^O - s) \right) \quad (8)$$

where $$\text{prob}(r_H) = \underbrace{(1-p)(1-\beta)b_L^*(1-g_H^*) + (1-p)\beta(1-g_H^*)}_{=\text{prob}(x_L, r_H)} + \underbrace{p\alpha b_L^* + p(1-\alpha)}_{=\text{prob}(x_H, r_H)}$$

and $$\text{prob(error)} = \text{prob}(y_L)b_L^*\left(1-g_H^*\right) f$$

Note that these probabilities indirectly depend on s through the equilibrium strategies b_L^* and g_H^*.

The manager accepts the contract offered by the owner if it meets her reservation utility, which we normalized with 0. Because compensation is also bound by 0, any contract yields nonnegative expected compensation. The crucial constraint is the manager's incentive constraint that ensures she chooses the high effort a_H. Recall that the effort choice occurs before the accounting system reports the signal y. The manager's expected utility is

$$E[U^M | a_H] = \text{prob}(r_H)s - V - \text{prob}(y_L)\frac{v}{2}b_L^{*2} - \text{prob}(y_L)b_L^*(1-g_H^*) fs \quad (9)$$

where the first term is the expected bonus, the second term, V, is the disutility of high effort, the third term is the expected cost of earnings management, and the fourth term is the expected claw-back of the bonus if the enforcer identifies an error. Substituting for prob(r_H) and b_L^*, the expected utility becomes

$$E[U^M|a_H] = s\,\text{prob}(y_H)\left(1 - g_H^* \,\text{prob}(x_L|y_H)\right) + \text{prob}(y_L)\frac{1}{2}vb_L^{*2} - V$$

The incentive compatibility constraint is

$$E[U^M|a_H] \geq E[U^M|a_L] = s\,\text{prob}(y_H|a_L)\left(1 - g_H^* \,\text{prob}(x_L|y_H, a_L)\right) + \text{prob}(y_L|a_L)\frac{1}{2}vb_{LL}^{*2} \qquad (10)$$

where $b_{LL}^* \equiv b_L(g_H^*|a_L)$ denotes the manager's adjusted earnings management effort if she deviated from the equilibrium production effort a_H. The auditor still conjectures a_H and b_L^*; hence, he does not adjust the equilibrium audit strategy g_H^*. Therefore, b_{LL}^* is based on the reaction function b_L, anticipating $\hat{g}_H = g_H^*$, which results in

$$b_{LL}^* = \frac{s}{v}[(1-f) + g_H^*\left(\text{prob}(x_H|y_L, a_L) - (1-f)\right)]$$

The right-hand side of (10) is always positive for $s > 0$, implying that a contract that satisfies incentive compatibility induces rents to the manager and thus clearly meets her reservation utility of 0.

After deviating from a_H to a_L, the manager would reduce earnings management because it becomes less likely that $x = x_H$. The probabilities are:

$$\text{prob}(x_H|y_L) = \frac{p\alpha}{p\alpha + (1-p)(1-\beta)} > \frac{q\alpha}{q\alpha + (1-q)(1-\beta)} = \text{prob}(x_H|y_L, a_L)$$

for $p > q$, which results in $b_{LL}^* < b_L^*$. However, the probability y_L increases and so do the instances of earnings management. Denote the minimum s that satisfies the incentive compatibility constraint (10) by $\underline{s} > 0$. The following proposition characterizes the optimal compensation contract.

***Proposition* 2**: Under mild conditions, the optimal bonus is determined by the manager's incentive compatibility constraint only, i.e., $s^* = \underline{s}$.

As shown in the appendix, $\underline{s}$ is implicitly defined by

$$\underline{s} = \frac{1}{(p-q)\left(1 - \alpha - \beta(1-g_H^*)\right)}\left[V + \frac{v}{2}\left(\text{prob}(y_L|a_L)b_{LL}^{*2} - \text{prob}(y_L)b_L^{*2}\right)\right]$$

The proof examines each cost component included in the owner's expected utility and establishes that the audit fee and the owner's expected cost of enforcement unambiguously increase in s. It also finds that the expected compensation (net of claw-back) increases in s under mild conditions. Together, these results imply that the owner chooses the bonus payment that just satisfies the incentive compatibility constraint, but does not pay more. The reason why formally mild conditions are required is subtle. Note that one would conjecture that an increase in s over $\underline{s}$ cannot be desirable to the owner, because it is not useful to increase productive effort but only increases the manager's earnings management incentives. This intuition holds for all (direct and indirect) effects of increasing s over and above $\underline{s}$, except for one effect: The probability that the manager receives the bonus, $\text{prob}(r_H) - \text{prob}(\text{error})$, directly depends on the audit effort g_H^*, which improves the quality of the financial report by reducing prob(r_H) through lowering the β-error. *Ceteris paribus*, an increase in s increases the audit effort, which reduces the probability of paying a bonus in a situation in which the productive outcome is x_L, but the accounting system reports y_H. The proof shows that this effect has a value of $(1-p)\beta \frac{dg_H^*}{ds} s$. It is small and most likely outweighed by the other effects that increase the owner's expected utility from increasing s^* over $\underline{s}$. Sufficient conditions, for example, are the following: β is „low,“ p is „high," or C^O is "high." Then the owner chooses the lowest s that implements a_H, which is $s^* = \underline{s}$. But it is impossible to formally exclude a case that this effect might dominate. In the subsequent analysis, we assume that the mild conditions stated in Proposition 2 are satisfied.

To conclude the analysis of the owner's decision problem, we consider what happens if it becomes too costly to the owner to induce the manager to provide high productive effort a_H. The next result provides the lower bound on the owner's expected utility.

***Lemma* 4**: The owner's expected utility from inducing a_L is

$$E[U^O \,|\, a_L] = (1-q)x_L + qx_H \tag{11}$$

Note that to induce a_L, the optimal contract pays the minimum compensation, which is $s(r_L) = s(r_H) = 0$. This compensation is independent of the financial report, which eliminates incentives of the manager to engage in earnings management – it would be costly, but of no benefit. The manager's expected utility for low productive effort a_L is 0. Enforcement will not find an error because there is no earnings management; hence, there is no cost of enforcement. Finally, the auditor has no incentive to provide audit effort either ($g_i = 0$). That is, $r_i = y_i$. In equilibrium, the auditor chooses $g_i = 0$ and expects no cost of enforcement. In a competitive market, the audit fee offered therefore is

$$A = \text{prob}\left(m_H\right)\left(\frac{k}{2} g_H \left(2 - g_H\right)\right) = 0$$

The expected outcome from the production process is higher for a_H than for a_L because

$$\left((1-p)x_L + px_H\right) - \left((1-q)x_L + qx_H\right) > 0$$

holds because $p > q$. This benefit comes at a higher cost of inducing a_H. Clearly, if the financial reporting system (and the institutional safeguards) is not sufficiently informative to use it for compensation purposes, the expected cost of inducing a_H can outweigh the expected benefit. For example, low (or no) enforcement may be such a case; increasing the level of enforcement then has a productive effect if it becomes beneficial to the owner to induce high effort. Our subsequent results show how the owner's expected utility varies with a change in the enforcement effectiveness. If the expected utility decreases for a change in enforcement, production becomes more costly and perhaps even too costly to sustain high productive effort.

4.2. Effects of enforcement on firm value

In this subsection, we examine how a change in enforcement effectiveness affects the management incentives provided by the owner and the expected utility of the owner, which is equivalent to the value of the firm in our setting.

The incentive compatibility constraint implicitly defines the minimum bonus,

$$\underline{s} = \frac{1}{(p-q)\left(1-\alpha-\beta(1-g_H^*)\right)}\left[V + \underbrace{\frac{v}{2}\left(\text{prob}(y_L\,|a_L)b_{LL}^{*2} - \text{prob}(y_L)b_L^{*2}\right)}_{\equiv D}\right] \tag{12}$$

The bonus $\underline{s}$ must be set sufficiently high to cover the manager's cost of effort V and the difference in (net) utility arising from the fact that the manager chooses the conditionally optimal earnings management effort given a_H and a_L, respectively (which is captured in the term D in (12)). These two costs are scaled by the factor $\frac{1}{(p-q)\left(1-\alpha-\beta(1-g_H^*)\right)}$, which captures the informativeness of the financial report r about the productive effort. Note that higher audit effort g_H^* reduces the required s because the auditor detects x more often, and this reflects a direct benefit of auditing on incentives.

The functional behavior of the second term is complex because it depends on two different earnings management strategies, one played in equilibrium (b_L^*) and the other out of equilibrium (b_{LL}^*). In general, equation (12) for $\underline{s}$ cannot be explicitly solved. To gain some insight, we consider the boundary cases $f = 0$ (no enforcement) and $f = 1$ (perfect enforcement). If $f = 0$, then the audit effort $g_H^* = 0$ and earnings management is equally high for both effort levels (i.e., $b_{LL}^* = b_L^*$). The low signal y_L occurs more frequently under a_L than under a_H because $\text{prob}(y_L | a_L) > \text{prob}(y_L)$, hence, the manager receives greater expected utility from earnings management if she chose the low effort. Therefore, $D(f = 0) > 0$. To be incentive compatible, the bonus must compensate the manager for the loss in expected benefits from earnings management if she decides to exert the high effort, but this increase in s in turn increases the earnings management incentive further. If $f = 1$, there is no earnings management, in which case $D(f = 1) = 0$, and D can be either positive or negative for f somewhat below $f = 1$.[13] The following result summarizes general properties of the minimum bonus $\underline{s}$, which is the optimal bonus under the conditions described in Proposition 2.

***Proposition* 3**: The minimum bonus $\underline{s}$ has the following properties:

(i) If $f = 0$, $\underline{s} > \dfrac{V}{(p-q)(1-\alpha-\beta)}$ and strictly decreases in f.

(ii) If $f = 1$, $\underline{s} = \dfrac{V}{(p-q)(1-\alpha-\beta)}$ and increases if f approaches 1 from below; the increase is strict if $\beta > 0$.

(iii) $\underline{s}$ attains a minimum for $f = f_1 \in (0, 1)$ and $\underline{s}(f_1) < \dfrac{V}{(p-q)(1-\alpha-\beta)}$ if $\beta > 0$.

[13] For example, D becomes negative if both α and β are close to 1/2.

The proof is in the appendix. Proposition 3 establishes that introducing enforcement has a non-monotonic effect on the optimal expected compensation: Increasing enforcement is beneficial for low levels of f, but becomes strictly detrimental for high levels of f (except in the case of $\beta = 0$). We discuss the intuition for this result below.

The bonus to induce the manager to exert high effort under $f = 0$ is strictly higher than that under perfect enforcement ($f = 1$); the required bonus in the latter case is $\underline{s} = \frac{V}{(p-q)(1-\alpha-\beta)}$, which is equal to the bonus that would result if the manager has no earnings management opportunity. In that case, enforcement would not identify any earnings management and the auditor would not exert audit effort because there is no risk of an enforcement action. This bonus is solely governed by the characteristics of the production technology and the accounting system. In particular, $\underline{s}$ decreases the more precise the accounting signal is (lower α and β).

The optimal bonus in case of no enforcement is strictly greater because the manager engages in earnings management $(b_L^* > 0)$, which is costly; and the differential between earnings management under productive effort levels a_H relative to a_L must be compensated by a higher bonus to continue to induce a_H. This increase in the bonus amplifies the earnings management incentive, which again pushes the required bonus further upwards.

Increasing f from $f = 0$ has the following effects: It introduces a risk of an enforcement action, which mitigates the incentive of the manager to manage earnings (due to the risk of a claw-back of the bonus) and induces the auditor to exert positive audit effort – this audit effort further mitigates earnings management in equilibrium. Both effects together increase the information content of the accounting report, which allows the owner to reduce the bonus, which further alleviates earnings management and audit effort somewhat until an optimum is reached. Proposition 3 (i) establishes that the total effect from increasing f from $f = 0$ strictly reduces the required bonus.

Proposition 3 (ii) shows that higher enforcement effectiveness increases the required bonus $\underline{s}$ if f increases to a value close to 1. Statements in (i) and (ii) together imply that the bonus $\underline{s}$ is minimal for a specific $f_1 \in (0, 1)$ and that this minimum is less than $\underline{s} = \frac{V}{(p-q)(1-\alpha-\beta)}$ (except for the knife-edge case of $\beta = 0$, in which $f_1 = 1$).

These characteristics suggest that the typical behavior of the optimal bonus (and the expected compensation cost) is u-shaped. The main reason that "too" strong enforcement is harmful for incentives is that enforcement substitutes audit effort if enforcement is strong, whereas it is a complement if enforcement is weak. Crowding out audit effort reduces the information content of reported earnings because it is the auditing function that uncovers and corrects errors that arise from the accounting system. Enforcement controls earnings management in the financial report (as does more auditing), but it is less useful than an audit due to its limited scope. While we assume that enforcement is costless to the firm, factoring in a cost of enforcement amplifies this disadvantage.

The owner's expected utility consists of the expected outcome less the expected bonus payment $\underline{s}$ (net of a potential claw-back), the audit fee A, and the expected cost of an enforcement action. The equilibrium audit fee is

$$A = \text{prob}(m_H)\frac{k}{2} g_H^*(2 - g_H^*) \tag{13}$$

which is directly increasing in k and equals 0 if $g_H^* = 0$, which is the case if $f = 0$ or 1. The owner's expected enforcement cost is

$$\text{prob}(y_L) b_L^*(1 - g_H^*) f C^O \tag{14}$$

which is linearly increasing in the cost of an enforcement action C^O and is 0 if $b_L^* = 0$, which is again the case if $f = 0$ or 1. Therefore, in the boundary cases of $f = 0$ and $f = 1$ the owner's expected utility equals the expected outcome minus the expected bonus payment, for which the relation in Proposition 3 holds. The following result summarizes the effects.

***Proposition* 4**: The owner's expected utility (firm value) is strictly greater under perfect enforcement ($f = 1$) than under no enforcement ($f = 0$). Varying enforcement effectiveness f within 0 and 1 can increase or decrease the owner's expected utility, depending on the parameters.

A reason for the indeterminate effects of varying $f \in (0, 1)$ is that the audit fee A is directly related to the audit cost parameter k (whereas the audit strategy and minimum bonus $\underline{s}$ only depend on the auditor's enforcement cost relative to the audit cost, C^A/k) and that the owner's enforcement cost depend directly on C^O. Therefore, varying these parameters directly affects the owner's expected utility. We illustrate the possible effects by an example using the following parameters: $p = 0.8$, $q = 0.2$, $\alpha = 0.2$, $V = 1$, $v = 40$, $C^A/k = 10$, $C^O = 1$; β takes values between 0 and 0.3, and k is either 1 or 5.[14] Figure 4 depicts the equilibrium earnings management and audit effort for the full range of enforcement effectiveness for $\beta = 0.1$. Equilibrium earnings management b_L^* always decreases for an increase in enforcement f, whereas equilibrium audit effort g_H^* first increases and then decreases for higher f. This illustrates the crowding-out effect of stronger enforcement on audit effort.

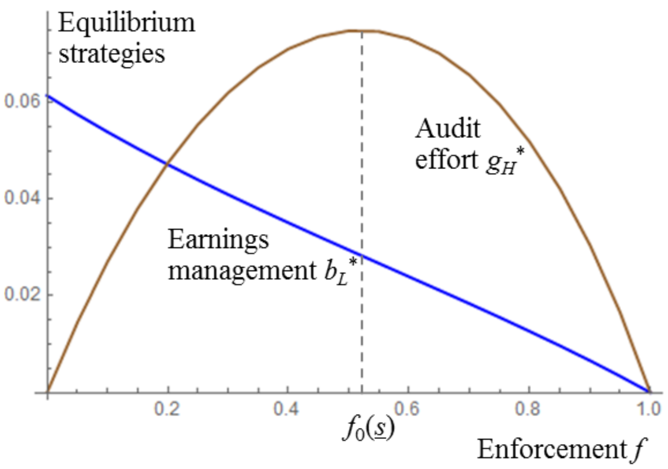

Figure 4: Equilibrium strategies under the optimal contract ($\beta = 0.1$)

Figure 5 plots the required bonus $\underline{s}$ for a variation of the enforcement for $\beta = 0$, 0.1, 0.2, and 0.3. A lower β is always beneficial to the owner because it makes the accounting system

[14] We keep C^A/k constant to ensure that equilibrium earnings management and audit effort are not affected by the change in k. That means that C^A is 10 and 50, respectively. Despite $C^A/k = 10$ does not satisfy the sufficient condition ($C^A/k \leq 1$) it ensures $g_H^* < 1$ in our examples.

more precise (*ceteris paribus*), which allows the owner to reduce the required bonus. $\beta = 0$ is the special case in which the bonus decreases in f over the full range of f, so that $f = 1$ minimizes the required bonus. For $\beta > 0$, the bonus minimizing enforcement effectiveness is strictly less than 1.

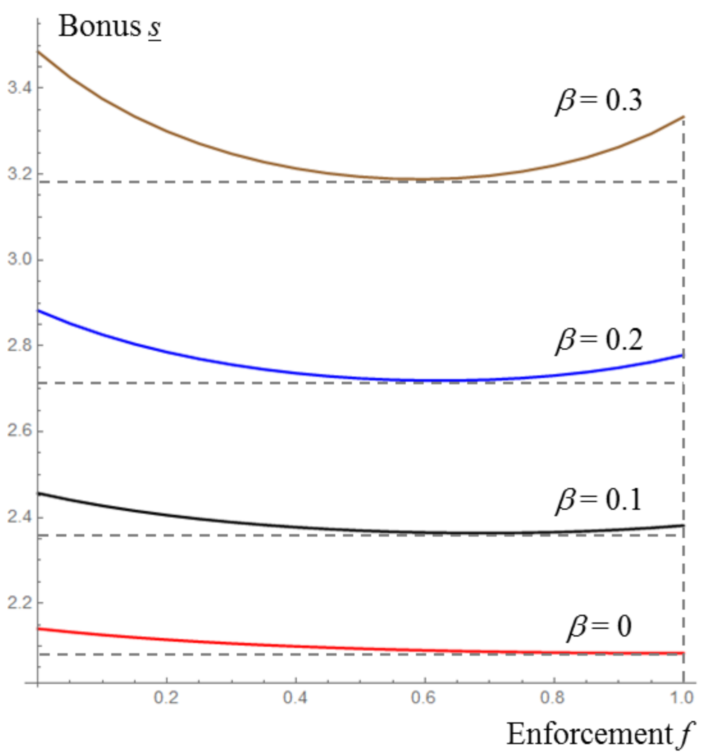

Figure 5: Optimal bonus for different values of β

Figure 6 depicts the expected utility to the owner, which reflects the owner's expected utility (firm value) before adding the constant expected outcome. Again, the owner's expected utility is greater the more precise the accounting system is (lower β) and, as stated in Proposition 4, it is higher under perfect enforcement ($f = 1$) than under no enforcement ($f = 0$). The effect of increasing enforcement f depends on the parameter constellations. In Figure 6, we vary k and C^A to show that for weak enforcement, increasing enforcement can either increase or decrease the owner' expected utility, and a similar functional behavior occurs for strong enforcement. Notice that for $\beta = 0.3$, $k = 1$ and $C^A = 10$, the expected cost is minimal at an enforcement level that is strictly less than perfect enforcement, suggesting that "too" much enforcement destroys firm value. While not shown in the Figure, a higher cost of an enforcement action C^O directly reduces the owner's utility in the range of $f \in (0, 1)$. A variation of C^O therefore" "convexifies" the owner's expected utility function.

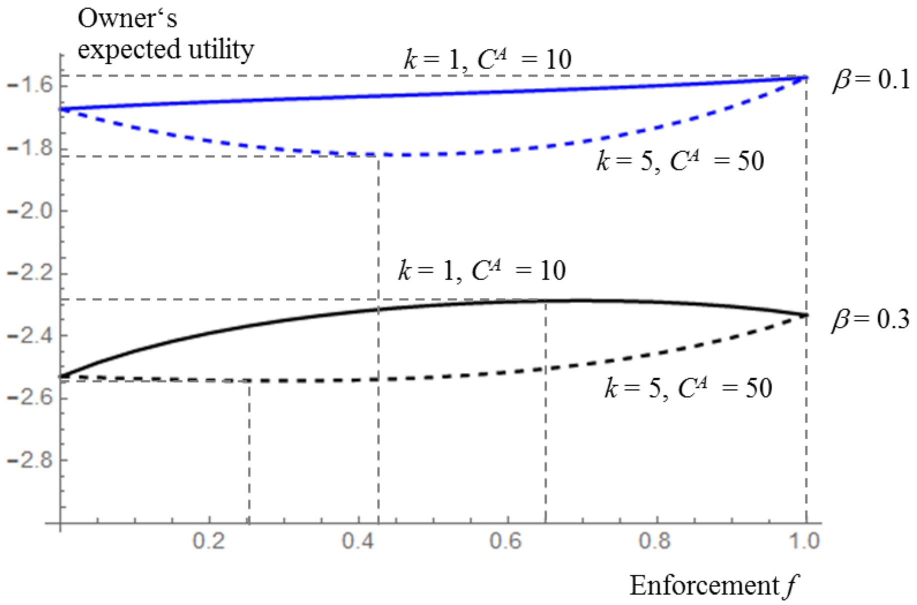

Figure 6: Owner's expected utility for different parameters

Finally, enforcement can have an immediate productive effect if the cost to induce a high productive effort a_H becomes so high that the owner is better off inducing the low productive effort a_L. In Figure 6 the latter option would introduce a constant line, $E[x|a_L]$ – $E[x|a_H]$, which can be greater or less than the expected cost curves. For example, consider the case with $\beta = 0.1$, $k = 5$, and $C^A = 50$: If $E[x|a_L] - E[x|a_H] = -1.75$, then if enforcement effectiveness is between [0, 0.12] or between [0.73, 1] the owner implements a_H, otherwise a_L. Therefore, if enforcement effectiveness was 0.1 and increases to 0.2, there is a loss in productivity.

5. Financial reporting quality

In this section, we examine the equilibrium financial reporting quality as a function of enforcement effectiveness f. Our measure of the quality of the audited financial report is the probability that the report r anticipates the ultimate outcome x, which captures the precision of the financial report. Financial reporting quality is

$$FRQ = 1 - \text{prob(divergence)} \tag{15}$$

A "divergence" occurs if the report differs from the final outcome, i.e., $r_i \neq x_i$ ($i = L, H$), which occurs with a probability of

$$\begin{aligned}\text{prob}\left(\text{divergence}\right) &= \text{prob}(r_L)\text{prob}(x_H \,|r_L\,) + \text{prob}(r_H)\text{prob}(x_L \,|r_H\,) \\ &= \text{prob}(x_H, r_L) + \text{prob}(x_L, r_H)\end{aligned}$$

The first term is the probability that the report understates the actual outcome,

$$\text{prob}(x_H, r_L) = p\alpha(1 - b_L)$$

and the second term is the probability that it overstates the outcome,

$$\begin{aligned}\text{prob}(x_L, r_H) &= (1-p)(1-\beta)b_L^*(1-g_H^*) + (1-p)\beta(1-g_H^*) \\ &= (1-p)(1-g_H^*)\left(\beta + b_L^*(1-\beta)\right)\end{aligned}$$

We focus our analysis on the unweighted sum of the two errors, but acknowledge that the cost of an under- or overstatement varies with the decision problem in which the financial report is used. Different weights do not qualitatively affect our results. Note that in our previous analysis of the owner's utility, the weights on different types of errors are determined endogenously for a stewardship purpose.

Rearranging terms, the total probability of a diverging report can be expressed through three terms, which facilitate to understand the sources for the errors:

$$\text{prob}\left(\text{divergence}\right) = \underbrace{p\alpha + (1-p)\beta}_{\equiv E_1 > 0} + \underbrace{b_L^*\left((1-p)(1-\beta) - p\alpha\right)}_{\equiv E_2} - \underbrace{(1-p)g_H^*\left(\beta + b_L^*(1-\beta)\right)}_{\equiv E_3 \geq 0} > 0 \tag{16}$$

The first term, E_1, is the *ex ante* probability of an α- and β-error that define the precision of the accounting system. This error is independent of earnings management, auditing, and enforcement.

The second term, E_2, represents the direct effect of earnings management on the probability of divergence. The sign of E_2 depends on the parameters of the accounting system. Note that the *ex ante* probability of a report y_L,

$$\text{prob}(y_L) = p\alpha + (1-p)(1-\beta)$$

is the sum of two events: $(1 - p)(1 - \beta)$ is the probability that $x = x_L$ and $y = y_L$, which is a correct depiction of the outcome, and $p\alpha$ is the probability that $x = x_H$ and the accounting system wrongly reports $y = y_L$. If the manager engages in earnings management, $b_L^* > 0$, then if successful, she reports r_H. If $x = x_L$, then earnings management disguises the originally correct signal y_L, which adds to the errors in the financial report. This is an instance of "bad" earnings management. Conversely, if $x = x_H$, then the accounting signal was wrong, and earnings management effectively corrects this wrong signal, which is "good" earnings management because it lowers the errors in the financial report. If

$$p\alpha > (1-p)(1-\beta) \tag{17}$$

then earnings management is "good" on average, otherwise it is "bad." Condition (17) is more likely to hold for greater p and for greater α and β.[15] That is, the less precise the accounting system is, the more does earnings management correct it. At the same time, a decrease in accounting precision implies an increase in $\text{prob}(x_H | y_L)$, the conditional probability that the high outcome actually obtains although the accounting system has produced the low signal. Considering the definition of T in (6), it is apparent that the presence of "good" earnings management and a positive relation between earnings management and (anticipated) audit effort are closely related. Given f, the less precise the accounting system, the higher is $\text{prob}(x_H | y_L)$ and the more likely it is that $T > 0$ holds, implying that a larger audit effort induces higher earnings management.

The third term in (16), E_3, captures the effect of auditing, which always leads to a (weak) reduction in the probability of divergence. It arises if the actual outcome is x_L (probability $1 - p$), but the accounting system produces a signal y_H because of the β-error and

[15] Notice this condition does not imply that a high α-error is desirable because $(E_1 + E_2)$ can increase or decrease in α. It only says that if $E_2 < 0$, an increase in earnings management reduces (*ceteris paribus*) the probability of an error and increases financial reporting quality.

earnings management. $E_3 = 0$ for the boundary cases of no enforcement ($f = 0$) and perfect enforcement ($f = 1$) because then $g_H^* = 0$.

Although f does not directly appear in the probability of divergence in (16), it affects earnings management and the audit effort and thus has an impact on earnings quality. Furthermore, the equilibrium earnings management b_L^* and the equilibrium audit effort g_H^* depend on the required bonus $\underline{s}$, which makes the effect of a variation of f complex. The following result provides some general insights.

***Proposition* 5**: Enforcement effectiveness f has the following effects on financial reporting quality *FRQ*:

(i) If enforcement is perfect ($f = 1$), then $FRQ(f = 1) = 1 - \left(p\alpha + (1-p)\beta\right)$.

(ii) $FRQ(f = 1) > FRQ(f = 0)$ if and only if $p\alpha < (1-p)(1-\beta)$.

(iii) *FRQ* is not necessarily monotonic in f.

Proposition 5 (i) first states that *FRQ* under perfect enforcement is simply the *FRQ* that arises from the accounting system itself, which is the *ex ante* expected error. Clearly, with perfect enforcement there is no earnings management and no auditing effort in equilibrium; hence, enforcement cannot identify any errors. Collectively, no errors in the accounting system are corrected.

The second result shows that perfect enforcement can lead to greater or less *FRQ* than no enforcement at all. This result contrasts with the result for firm value in Proposition 4, where we record the result that firm value is always strictly greater for perfect than for no enforcement. We also note that financial reporting quality is unaffected by several parameters that influence firm value, such as the productive probability q if the manager chooses the out-of-equilibrium action a_L, the auditor's cost k and C^A that enter *FRQ* only by the aggregate C^A/k, and the owner's cost of enforcement action C^O. Varying any of these parameters automatically induce different behaviors of *FRQ* and firm value.

The condition for whether $FRQ(f = 1)$ is greater or less than $FRQ(f = 0)$ is whether earnings management is "bad" or "good." To see this, recall that

$$\text{prob}\left(\text{divergence}\right) = \underbrace{p\alpha + (1-p)\beta}_{\equiv E_1 > 0} + \underbrace{b_L^*\left((1-p)(1-\beta) - p\alpha\right)}_{\equiv E_2} - \underbrace{(1-p)g_H^*\left(\beta + b_L^*(1-\beta)\right)}_{\equiv E_3 \geq 0}$$

E_1 is constant and for $f = 1$ we have $b_L^* = 0$ and $g_H^* = 0$, whereas for $f = 0$ we have $b_L^* > 0$ and $g_H^* = 0$. That is, $E_3 = 0$ in both cases. E_2 is greater than zero if and only if $p\alpha < (1-p)(1-\beta)$, that is, earnings management is "bad" on average, and vice versa.

Proposition 5 (iii) states that *FRQ* is not necessarily monotonic in *f*, which we show by some numerical examples because the actual functional form of *FRQ* depends on several parameters. Figure 7 depicts the equilibrium financial reporting quality for the same example as in Figure 5 for $\beta = 0$, 0.1, 0.2, and 0.3 to show the different behaviors of enforcement changes on decision usefulness and stewardship. The other parameters are: $p = 0.8$, $q = 0.2$, $\alpha = 0.2$, $V = 1$, $v = 40$, $C^A/k = 10$. Naturally, *FRQ* is higher for lower errors in the accounting system, captured by β in this example. The case $\beta = 0$ is a special case in which *FRQ* always increases. $\beta = 0.2$ is the special case in which $E_2 = 0$ (i.e., earnings management is informationally neutral on average) and shows that in this case *FRQ*($f = 0$) = *FRQ*($f = 1$). $\beta = 0.1$ is a case of "bad" earnings management, whereas $\beta = 0.3$ is a case of "good" earnings management. In these examples *FRQ* behaves inversely u-shaped, i.e., increases in *f* for low *f* and decreases for high *f*.

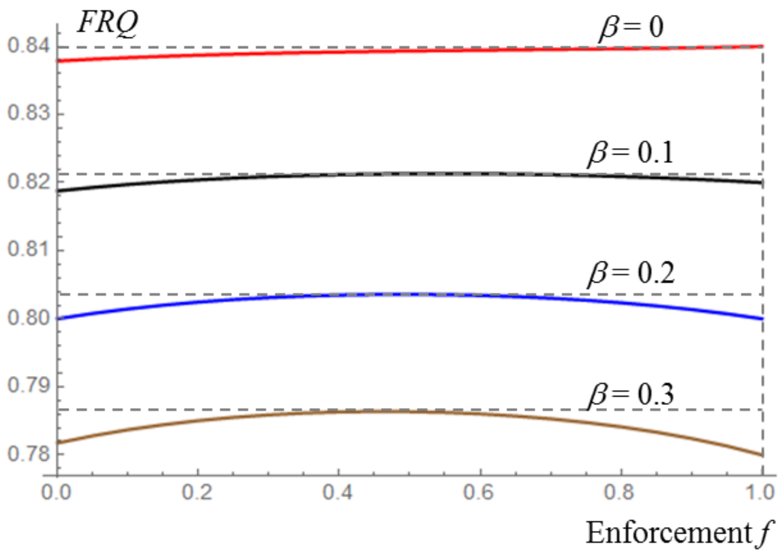

Figure 7: Equilibrium financial reporting quality for different values of β

Consider $p = 0.9$, $\alpha = 0.2$, $\beta = 0.2$, $V = 1$, $v = 20$, and $C^A/k = 1$ next. This case exhibits strong "good" earnings management. Figure 8 shows that the total divergence strictly increases with higher f, which means that *FRQ* strictly decreases with stronger enforcement, regardless of the original level of enforcement. Moreover, it shows that this effect results from the "good" earnings management that is depicted in the error E_2; E_1 provides the base level of error from the accounting system, and E_3 has little dampening effect in this particular example. Again, this result is in strong contrast to the effect of enforcement on firm value, which always increases at least over some interval of enforcement levels.

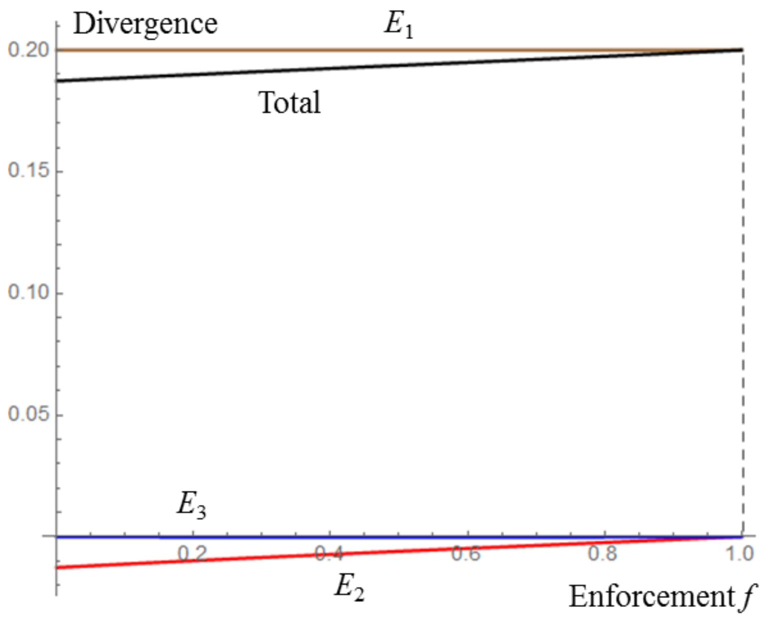

Figure 8: Probability of divergent audited financial report
E_1 = Effect of accounting system (*ex ante* probability of error)
E_2 = Effect of earnings management
E_3 = Effect of audit

It is also interesting to examine the information effect of an enforcement action in our model. As enforcement results are published only after a lengthy investigation, the information contained in the announcement of an enforcement is not very useful to learn about x, but more generally is informative about a firm's accounting system and behaviors if these are uncertain (which we do not model). With this caveat, note that the enforcer states an error only in case the report is r_H, the auditor fails to learn the outcome x, and the enforcer discovers $y = y_L$, which occurs with a probability of

$$\text{prob(error)} = \text{prob}(y_L) b_L^* \left(1 - g_H^*\right) f \tag{18}$$

This probability captures two distinct events: (i) An enforcement action leads to a correction of a deviation of the financial report if the report is r_H, the enforcer observes $y = y_L$, the auditor did not learn x, and the outcome is in fact x_L, which occurs with probability

$$(1-p)(1-\beta) b_L^* (1-g_H^*) f$$

A restatement in this case unambiguously increases financial reporting quality. (ii) However, enforcement itself is not free of error because it does not uncover the outcome x_i but only the accounting signal y_j that provides imprecise information about x.[16] In this case, the enforcer states an error even though the audited financial report was correct. This event occurs if the auditor did not learn x, but $x = x_H$, because then the enforcer's alleged error cannot be challenged by audit evidence. The probability of this event is

$$\text{prob}(y_L) b_L^* \left(1 - g_H^*\right) f \, \text{prob}(x_H | y_L) = p\alpha b_L^* \left(1 - g_H^*\right) f$$

and a restatement decreases financial reporting quality.

The net change of prob(divergence) is

$$\begin{aligned} E_4 &\equiv -(1-p)(1-\beta) b_L^* (1-g_H^*) f + p\alpha b_L^* (1-g_H^*) f \\ &= f(1-g_H^*) b_L^* \left(p\alpha - (1-p)(1-\beta)\right) \end{aligned}$$

Note that $E_4 = -f(1-g_H^*)E_2 < -E_2$, so the net effect of the enforcement action mitigates the effect of E_2 on *FRQ*. It is easy to see that the announcement of an enforcement action increases *FRQ* only if $p\alpha < (1-p)(1-\beta)$, that is, earnings management is "bad."

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[16] Another error occurs if the enforcer does not state an error, although there is in fact one. This occurs if the enforcer does not learn y, and the resulting error is embedded in the probability of a deviating report, which we analyze earlier.

6. Robustness

Our model rests on a number of simplifying assumptions to facilitate tractability. We believe that relaxing most assumptions does not qualitatively affect the results we establish because the main strategic interactions between the players appear robust. To examine the robustness or our results, we discuss the effects of changes of key assumptions.

A fundamental assumption is that enforcement activities differ from audit services, that is, enforcement is not simply a second full audit. We capture this difference by assuming that the auditor observes (x, y) whereas the enforcer only observes y. Our results should extend to a situation in which the auditor alternatively observes some additional imperfect signal about x. An imperfect accounting system is also a driving force for our findings. For example, assume a perfect accounting system ($\alpha = \beta = 0$); then enforcement has always a positive effect because earnings management is always "bad" and no earnings management leads to fully revealing financial reports.

We assume throughout that, if the enforcer finds an error $r_i \neq y_i$, the auditor can convince the enforcer to accept the evidence x to support r_i, which in this case is $r_i = x_i$, $i = L, H$. There may be reasons to assume that the enforcer does not withdraw the error allegation and initiates an enforcement action. For example, the enforcer may favor full compliance with the accounting standards, so that earnings management (even if it is "good") is abandoned. Alternatively, the auditor may incur a significant cost to present the evidence, and this cost may be prohibitive; or the auditor does not always uncover x, but may only find out y (as does the enforcer).

Our main insights do not significantly change with such alternative assumptions. To see what results are affected, assume that the enforcer will always trigger an enforcement action if it finds that $r_i \neq y_i$. The manager's optimal bias does no longer depend on $\text{prob}(x_H | y_L)$ but equals

$$b_L = \frac{s}{v}(1 - \hat{g}_H)(1 - f)$$

This bias is smaller than under the original assumption and strictly decreases in the conjectured audit effort and the enforcement effectiveness. Because expectations about the true *x* do not matter, the optimal bias is independent of the productive action, i.e., $b_L = b_{LL}$. The expression for the optimal audit is formally unchanged, but the optimal audit effort becomes smaller as well. The results in Proposition 1 and Corollary 1 continue to hold (the only difference is that $T = -(1-f) \leq 0$). In particular, higher enforcement still crowds out audit effort.

However, auditing becomes less beneficial because the enforcement overrides the corrective effect of audit findings since *x* becomes irrelevant. As a consequence, the crowding-out becomes less detrimental for the owner. Consider the new incentive compatibility, which determines the optimal compensation,

$$\begin{aligned}
\underline{s} &= \frac{1}{(p-q)(1-\alpha-\beta)}\left[V + \frac{v}{2}\left(\text{prob}(y_L|a_L)b_{LL}^{*2} - \text{prob}(y_L)b_L^{*2}\right)\right] \\
&= \frac{1}{(p-q)(1-\alpha-\beta)}\left[V + \frac{v}{2}b_L^{*2}\left(\text{prob}(y_L|a_L) - \text{prob}(y_L)\right)\right] \\
&= \frac{1}{(p-q)(1-\alpha-\beta)}\left[V + \frac{v}{2}b_L^{*2}(p-q)(1-\alpha-\beta)\right] \\
&= \frac{V}{(p-q)(1-\alpha-\beta)} + \frac{v}{2}b_L^{*2}
\end{aligned}$$

It is similar to that under the original assumption for the boundary cases $f = 0$ and $f = 1$, and $\underline{s}$ now strictly decreases in *f*.

Finally, consider the effect of the alternative assumption on financial reporting quality. According to (16) the probability of divergence is equal to

$$\begin{aligned}
\text{prob}(\text{divergence}) &= \underbrace{p\alpha + (1-p)\beta}_{\equiv E_1 > 0} + \underbrace{b_L^*\left((1-p)(1-\beta) - p\alpha\right)}_{\equiv E_2} - \underbrace{(1-p)g_H^*\left(\beta + b_L^*(1-\beta)\right)}_{\equiv E_3 \geq 0} \\
&= E_1 + b_L^*\left(1-g_H^*\right)(1-p)(1-\beta) - b_L^* p\alpha - (1-p)g_H^*\beta
\end{aligned}$$

The β-error is no longer corrected if the enforcer identifies an error, and the last term vanishes. Additionally, there is less correction of an α-error. Together, the probability of divergence becomes ($\bar{b}_L^*$ and $\bar{g}_H^*$ denote the optimal bias and audit effort under the alternative assumption)

$$\text{prob(divergence)} = E_1 + \bar{b}_L^*(1-\bar{g}_H^*)\left((1-p)(1-\beta) - p\alpha\right)$$

Therefore, if earnings management is "bad" on average (i.e., $(1-p)(1-\beta) - p\alpha > 0$), then the bias-induced increase of prob(divergence) is mitigated, but if earnings management is "good" the reduction of prob(divergence) through the manager's bias decreases. Because of $\frac{d\left(\bar{b}_L^*\left(1-\bar{g}_H^*\right)\right)}{df} < 0$,[17] *FRQ* strictly decreases in enforcement effectiveness for "good" earnings management (and vice versa), which is a more "extreme" result than that we find under the original assumption.

Another assumption is that the manager does not observe *x* (although the auditor does). Again, what is important for our results is that the auditor becomes better informed about *x* than the enforcer. Assume alternatively that the manager obtains the same information as the auditor, in our case *x*. If the manager learns that $x = x_H$, but $y = y_L$, the manager always wants the auditor to exert more effort because she knows that this will increase the probability of receiving a bonus; moreover, she would engage in more earnings management to correct this error in the accounting system. The reverse occurs if the manager learns $x = x_L$. This brief discussion suggests that earnings management becomes contingent on *x*, which adds an additional layer of complexity to our analysis.

We assume binary productive effort. This assumption simplifies the analysis because it keeps productive effort constant, until the cost of inducing high effort becomes so large that the owner shies away from providing any incentives. A continuous productive effort space would allow fine-tuning the desired effort, which again affects the equilibrium outcomes.

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[17] As shown in the proof of Proposition 2, we note that $b_L(1-g_H)$ is a strictly increasing function of b_L, and if the bias decreases in *f*, so does $b_L(1-g_H)$.

The manager is protected by limited liability. In an enforcement case, there are often non-financial sanctions in addition to penalties. Existence of such sanctions would make earnings management more costly to the manager, but as we show, this need not translate into greater firm value or financial reporting quality particularly if earnings management is "good."

We also assume that the incentive for the auditor to perform a quality audit stems from the risk that enforcement identifies an error. This assumption has two consequences: (i) If enforcement is perfect, which eliminates earnings management totally, the auditor has no incentive to provide audit effort, and (ii) anticipating that the manager tends to overstate earnings, the auditor has no incentive to audit low earnings. Because auditing is a value-adding service, less auditing reduces financial reporting quality. In reality, there are other mechanisms that impose incentives to auditors, such as audit inspections by an audit oversight body (such as the PCAOB) or auditor liability from litigation by parties that relied on the audited report. Such mechanisms also provide a strict preference for correcting misstatements even if the enforcer would not find them, such as errors in the accounting system and internal controls.

We model the enforcement institution as a "technology" because we believe an enforcer is mainly driven by the budget it has available and not by profit maximization. This means the enforcer does not act strategically and does not anticipate particular strategies by the manager or the auditor. However, persons responsible for enforcement may be loss averse or have other individual objectives, which then affect the enforcement strategy. Our model does not consider the threat of lawsuits by persons affected by financial reporting quality, which may affect the manager's or the auditor's strategies. These, as well as other, considerations provide avenues for future research.

7. Conclusions

This paper challenges the conventional wisdom that increasing enforcement of financial reporting has positive economic effects. This assumption ignores the fact that the strategies of the owner, managers, and auditors are interrelated and are determined in an equilibrium. We show that stronger enforcement, even if it is costless, can be detrimental for firm value and for the resulting financial reporting quality and we provide insights when this result arises.

We identify two reasons that are responsible that better enforcement can be detrimental: First, introducing enforcement increases audit effort, but if enforcement becomes sufficiently strong, it crowds out auditing. Because enforcement is more limited in scope than auditing, this crowding out effect diminishes financial reporting quality. This result is important because enforcement institutions often care more about compliance than about a fair presentation of firms' economics. We show that this focus on compliance can have detrimental effects for both firm value and financial reporting quality.

Second, earnings management is not necessarily "bad" but can be "good" if the accounting system erroneously understates earnings. A manager with earnings-based compensation has an incentive to manage earnings upwards, which adds bias if the misrepresentation overstates earnings, but improves financial reporting quality if it corrects an understatement. Stronger enforcement mitigates earnings management and if it is "good," financial reporting quality declines.

Empirical predictions

Our main result is that increasing enforcement can increase, but also decrease, firm value and financial reporting quality. The ambiguity of this result makes it difficult to directly test this result, aside from issues that arise from finding suitable proxies for each of the variables. One fundamental insight of the present paper is to caution that assuming stronger enforcement is always desirable is not true.

Another fundamental insight is that the effect of enforcement on firm value differs from that on financial reporting quality. Our results on financial reporting quality are more diverse in the sense that we find that quality can decrease over the full range of enforcement effectiveness. Standard setters, such as the FASB and the IASB, are mainly concerned with financial reporting quality. Our results stand in contrast to the Conceptual Frameworks by the FASB (2010) and the IASB (2010) that the objective of decision usefulness also encompasses stewardship. Of course, this insight is not novel in the theoretical literature. Gjesdal (1982) shows that in a more general agency model the ranking of accounting systems designed for different purposes do not coincide. However, that does not necessarily imply that the differences in the respective optimal accounting systems are large.[18] Our findings are in line with these results.

In the following, we state several empirical predictions that are either unambiguous or can be anchored on determining parameters.

(i) An increase of enforcement unambiguously mitigates earnings management.

(ii) Audit fees increase in enforcement effectiveness if the enforcement is weak, but decrease if the enforcement is strong.

(iii) Strengthening enforcement from an already high level is likely to reduce firm value, but can either increase or decrease financial reporting quality.

(iv) Generally, strengthening enforcement is more likely to increase firm value and financial reporting quality if the accounting system is more precise.

We hope these – and potentially other – predictions are helpful to design empirical tests to gain better insights into the interaction between auditing and enforcement and their joint effects on firm value and financial reporting quality.

[18] See, e.g., Drymiotes and Hemmer (2013).

References

Antle, R. (1982): The Auditor as an Economic Agent, *Journal of Accounting Research* 20, 503–527.

Baiman, S., J.H. Evans, and J. Noel (1987): Optimal Contracts with a Utility-Maximizing Auditor, *Journal of Accounting Research* 25, 217–244.

Ball, R., S.P. Kothari, and A. Robin (2000): The Effect of International Institutional Factors on Properties of Accounting Earnings, *Journal of Accounting and Economics* 29, 1–51.

Bertomeu, J., M. Darrough, and W. Xue (2015): Optimal Conservatism with Earnings Manipulation, Working Paper, City University of New York.

Brown, P., J. Preiato, and A. Tarca (2014): Measuring Country Differences in Enforcement of Accounting Standards: An Audit and Enforcement Proxy, *Journal of Business Finance and Accounting* 41, 1–52.

Christensen, H.B., L. Hail, and C. Leuz (2013): Mandatory IFRS Reporting and Changes in Enforcement, *Journal of Accounting and Economics* 56(2–3, Supplement 1), 147–77.

Deng, M., N. Melumad, and T. Shibano (2012): Auditors' Liability, Investments, and Capital Markets: A Potential Unintended Consequence of the Sarbanes-Oxley Act, *Journal of Accounting Research* 50, 1179-1215.

Drymiotes, G., and T. Hemmer (2013): On the Stewardship and Valuation Implications of Accrual Accounting Systems, *Journal of Accounting Research* 51, 281–334.

Ernstberger, J., O. Vogler, and M. Stich (2012): Economic Consequences of Accounting En-forcement Reforms: The Case of Germany, *European Accounting Review* 21, 217–251.

EU (2004): Transparency Directive, Directive 2004/109/EC, December 2004, available at: http://eur-lex.europa.eu/legal-content/EN/TXT/PDF/?uri=CELEX:02004L0109-20080320&from=EN.

Ewert, R. (1999): Auditor Liability and the Precision of Auditing Standards, *Journal of Institutional and Theoretical Economics* 155, 181–206.

Ewert, R., and A. Wagenhofer (2011): Earnings Management, Conservatism, and Earnings Quality, *Foundations and Trends in Accounting* 6, 65–186.

Feltham, G.A., and J. Xie (1994): Performance Measure Congruity and Diversity in Multi-Task Principal/Agent Relations, *The Accounting Review* 69, 429–453.

Fischer, P.E., and R.E. Verrecchia (2000): Reporting Bias, *The Accounting Review* 75, 229–245.

Gjesdal, F. (1982): Information and Incentives: The Agency Information Problem, *Review of Economic Studies* 49, 373–390.

Glover, J., and C. Levine (2015): Information Asymmetries about Measurement Quality, Working Paper, Columbia University.

Hillegeist, S.A. (1999): Financial Reporting and Auditing under Alternative Damage Apportionment Rules, *The Accounting Review* 74, 347–369.

Hope, O.-K. (2003): Disclosure Practices, Enforcement of Accounting Standards, and Analysts' Forecast Accuracy: An International Study, *Journal of Accounting Research* 41, 235–272.

Königsgruber, R. (2012): Capital Allocation Effects of Financial Reporting Regulation and Enforcement, *European Accounting Review* 21, 283–296.

Laux, C., and V. Laux (2009): Board Committees, CEO Compensation, and Earnings Management, *The Accounting Review* 84, 869-891.

Laux, V., and P.C. Stocken (2015): Accounting Standards, Regulatory Enforcement, and Innovation, Working Paper, University of Texas at Austin.

Pae, S., and S.-W. Yoo (2001): Strategic Interaction in Auditing: An Analysis of Auditors' Legal Liability, Internal Control System Quality, and Audit Effort, *The Accounting Review* 76, 333-356.

SEC (2000): Concept Release: International Accounting Standards, available at: https://www.sec.gov/rules/concept/34-42430.htm.

Shibano, T. (1990): Assessing Audit Risk from Errors and Irregularities, *Journal of Accounting Research* 28, Supplement, 110–140.

Smith, J.R., S.L. Tiras, and S.S. Vichitlekarn (2000): The Interaction between Internal Control Assessment and Substantive Testing in Audits for Fraud, *Contemporary Accounting Research* 17, 327–356.

Appendix

Summary of notation

a	Productive effort by manager, $a \in \{a_L, a_H\}$
A	Audit fee
b_j	Earnings management: probability of report m_i given y_j
C^O	Cost of enforcement action to owner
C^A	Cost of enforcement action to auditor
D	Term for earnings management part in manager's expected utility
E_i	Probability terms
f	Probability of enforcer to detect y
FRQ	Financial reporting quality (1 – probability of divergence of $r_i \neq x_i$)
g_j	Audit effort: probability of observing correct x given report m_j
k	Scaling factor of cost of audit effort
m	Preliminary report of manager, $m \in \{m_L, m_H\}$
p	Probability of high outcome x_H given high effort a_H
q	Probability of high outcome x_H given low effort a_L
r	Audited financial report, $r \in \{r_L, r_H\}$
s	Manager's compensation ($s(r_j)$), bonus paid for high earnings
T	Condition on probabilities
U^A	Utility of auditor
U^M	Utility of manager
U^O	Utility of owner
v	Scaling factor of disutility of earnings management b
V	Disutility of manager for a_H
x	Productive outcome, $x \in \{x_L, x_H\}$
y	Signal from accounting system, $y \in \{y_L, y_H\}$
α	„α-error", probability of report y_L given x_H
β	„β-error", probability of report y_H given x_L

Proofs

Proof of Lemma 1

A compensation with both $s(r_H)$ and $s(r_L) > 0$ cannot be optimal because the manager's reservation utility is 0 and compensation can be reduced by $\min\{s(r_L), s(r_H)\}$ without changing the manager's incentives, but increasing the owner's utility. That is, at least one of the compensation payments must be zero.

If both $s(r_H)$ and $s(r_L) = 0$ then the compensation does not depend on the financial report, which therefore becomes useless. The manager does not engage in earnings management because it is costly, and the enforcer will not find any error, hence, there is no cost of enforcement. The high effort a_H is not implementable because the manager's disutility is $V > 0$, but the expected compensation is the same for a_H and a_L. Therefore, two cases remain: $s(r_H) > s(r_L) = 0$ and $s(r_L) > s(r_H) = 0$.

Case 1: $s(r_H) > s(r_L) = 0$. The manager's utility conditional on y_H (gross of effort and enforcement costs) becomes

$$E[U^M | a_H, y_H] = \text{prob}(r_H | y_H) s(r_H) - \frac{1}{2} v b_H^2$$

where $\hat{g}_i$ denotes the conjectured audit effort and $\text{prob}(r_H | y_H) = (1-b_H)\big((1-\hat{g}_H) + \text{prob}(x_H | y_H)\hat{g}_H\big) + b_H \text{prob}(x_H | y_H)\hat{g}_L$. Differentiating $\text{prob}(r_H | y_H)$ with respect to b_H yields

$$\frac{\partial}{\partial b_H} \text{prob}(r_H | y_H) = \text{prob}(x_H | y_H)(\hat{g}_L - \hat{g}_H) - (1-\hat{g}_H)$$

A necessary condition for $b_H > 0$ is that this derivative is positive. However, this cannot be the case because $0 < \text{prob}(x_H | y_H) < 1$ and $\hat{g}_L < 1$ (recall the $\hat{g}_i$ are probabilities). Therefore, $b_H = 0$.

Case 2: $s(r_L) > s(r_H) = 0$. Due to symmetry, the same analysis applies for $y = y_L$, with a change in the indexes L and H. As a result, it must be the case that $b_L = 0$.

Next consider how the manager's expected utility changes in p. Recall that the audited financial report is as follows:

$$r_i = \begin{cases} x_i & \text{with probability } g_i \\ m_i & \text{with probability } (1-g_i) \end{cases}$$

Suppose the audit is ineffective, implying $r_i = m_i$. Rewriting the manager's expected gross utility yields

$$E[U^M | a_H] = (1-\text{prob}(m_H))\, s(r_L) + \text{prob}(m_H) s(r_H) - \frac{v}{2}\left(\text{prob}(y_L) b_L^2 + \text{prob}(y_H) b_H^2\right)$$

$$= s(r_L) + \left[b_L + \text{prob}(y_H)(1-b_L-b_H)\right]\left(s(r_H)-s(r_L)\right) - \frac{v}{2}\left(\text{prob}(y_L) b_L^2 + \text{prob}(y_H) b_H^2\right)$$

Because prob(y_H) > $\text{prob}(y_H | a_L)$, the expected utility must *ceteris paribus* increase in prob(y_H) to compensate for the higher disutility of $a_H > a_L$, that is,

$$\frac{\partial}{\partial \text{prob}(y_H)} E[U^M | a_H] = (1-b_L-b_H)\left(s(r_H)-s(r_L)\right) - \frac{v}{2}\left(b_H^2 - b_L^2\right) > 0$$

In case 1, $s(r_H) > s(r_L) = 0$ and $b_H = 0$, the derivative becomes

$$(1-b_L) s(r_H) + \frac{v}{2} b_L^2 > 0$$

In case 2, $s(r_L) > s(r_H) = 0$ and $b_L = 0$ it is

$$-(1-b_H) s(r_L) - \frac{v}{2} b_H^2 < 0$$

which contradicts the fact that $E[U^M]$ must increase in prob(y_H). Therefore, case 2 cannot be a feasible solution to the problem, which leaves case 1.

If the audit is perfect (that is, $r_i = x_i$) the only difference to the analysis is that prob(x_i) = p replaces prob(m_i). The conclusion is the same. The same analysis holds for any combination of x_i and m_i.

Finally, if the audit is perfect, there is no cost of enforcement. If $g_i < 1$, the manager incurs enforcement costs through a claw-back of a bonus only if r_H is reported. If the report is r_L and the enforcer finds out that $y = y_H$, there is no consequence to the manager because he did not receive a bonus for r_L. For a report of r_H, enforcement is tied to the probability of the enforcer finding an error. As shown in subsequent analyses, this probability increases in prob(y_L). Because prob(y_L) < $\text{prob}(y_L \mid a_L)$ the cost of enforcement is smaller for a_H, which establishes the Lemma. □

Proof of Lemma 2

If the auditor observes m_L, given his conjecture that the manager did not manage earnings ($\hat{b}_H = 0$, where the "hat" indicates the conjecture), the auditor correctly anticipates that the enforcer will never find or allege an error because $m_L = y_L$. Therefore, $\text{prob}(\text{error} \mid m_L) = 0$ and the auditor faces no cost of enforcement. Consequently, $g_L = 0$.

If the auditor observes m_H, there is the chance that the enforcer identifies an error, which occurs if the auditor does not find out x (so that $r = r_H = m_H$) and the enforcer learned $y = y_L$. The conditional probability of an error is (see again Figure 3)

$$\text{prob}(\text{error} \mid m_H) = \text{prob}(y_L \mid m_H)(1 - g_H) f$$

where $\text{prob}(y_L \mid m_H) = \dfrac{\text{prob}(y_L)\hat{b}_L}{\text{prob}(y_L)\hat{b}_L + \text{prob}(y_H)}$, which is greater 0 if $\hat{b}_L > 0$. The auditor's conditional expected utility is

$$U^A(m_H) = A - \frac{k}{2} g_H^2 - \text{prob}(y_L \mid m_H)(1 - g_H) f C^A$$

The first derivative with respect to g_H equals

$$\frac{\partial}{\partial g_H} U^A(m_H) = -k g_H + \text{prob}(y_L \mid m_H) f C^A$$

and setting it 0, the optimal audit effort is

$$g_H = \text{prob}(y_L \mid m_H) f \frac{C^A}{k} > 0$$

if $f > 0$. Our assumption that $C^A/k < 1$ ensures $g_H < 1$. □

Proof of Proposition 1

The manager maximizes her expected utility with respect to b_L,

$$E[U^M \mid a_H, y_L] = \left(\text{prob}(r_H \mid y_L) - b_L(1 - \hat{g}_H) f\right) s - V - \frac{1}{2} v b_L^2$$
$$= s b_L\left((1 - \hat{g}_H)(1 - f) + \hat{g}_H \, \text{prob}(x_H \mid y_L)\right) - V - \frac{1}{2} v b_L^2$$

The first order condition is

$$\frac{\partial}{\partial b_L} E[U^M \left| a_H, y_L \right.] = s\left((1-\hat{g}_H)(1-f) + \hat{g}_H \operatorname{prob}(x_H \left| y_L \right.)\right) - v b_L = 0$$

implying $$b_L = \frac{s}{v}[(1-f) + \hat{g}_H \underbrace{\left(\operatorname{prob}(x_H \left| y_L \right.) - (1-f)\right)}_{\equiv T}] \geq 0$$

We assume that v is sufficiently large to ensure $b_L < 1$. In the proof of Proposition 2, we show that the exact threshold we require is $v > \dfrac{2V}{(p-q)(1-\alpha-\beta)}$.

Existence and uniqueness of an equilibrium in the feasible range for b_L and g_H follows from a fixed point argument. b_L is strictly positive and linearly increasing in $\hat{g}_H$ if $T > 0$ and linearly decreasing otherwise. The boundaries are

$$b_L = \frac{s}{v}\left(\left((1-\hat{g}_H)(1-f) + \hat{g}_H \operatorname{prob}(x_H \left| y_L \right.)\right)\right) = \begin{cases} \frac{s}{v}(1-f) & \text{for } \hat{g}_H = 0 \\ \frac{s}{v}\operatorname{prob}\left(x_H \left| y_L \right.\right) & \text{for } \hat{g}_H = 1 \end{cases}$$

According to Lemma 2, $g_H = \operatorname{prob}(y_L \left| m_H \right.) f \dfrac{C^A}{k}$ with boundaries

$$g_H = \frac{\operatorname{prob}(y_L)\hat{b}_L}{\operatorname{prob}(y_L)\hat{b}_L + \operatorname{prob}(y_H)} \frac{fC^A}{k} = \begin{cases} 0 & \text{for } \hat{b}_L = 0 \\ \operatorname{prob}(y_L)\frac{fC^A}{k} & \text{for } \hat{b}_L = 1 \end{cases}$$

Note that g_H is strictly concave in $\hat{b}_L$ because

$$\frac{\partial g_H}{\partial \hat{b}_L} = \frac{\operatorname{prob}(y_L)\operatorname{prob}(y_H)}{\left(\operatorname{prob}(y_L)\hat{b}_L + \operatorname{prob}(y_H)\right)^2} \frac{fC^A}{k} > 0$$

$$\text{and} \quad \frac{\partial^2 g_H}{\partial \hat{b}_L^2} = -2\frac{\operatorname{prob}(y_L)^2 \operatorname{prob}(y_H)}{\left(\operatorname{prob}(y_L)\hat{b}_L + \operatorname{prob}(y_H)\right)^3} \frac{fC^A}{k} < 0$$

The equilibrium conditions are $\hat{b}_L = b_L$ and $\hat{g}_H = g_H$. The two reaction functions $b_L(\hat{g}_H)$ and $g_H(\hat{b}_L)$ are monotonic and continuous, hence, the function $g_H\left(b_L(\hat{g}_H)\right)$ is continuous, too. Furthermore, we have $0 \leq \hat{b}_L, \hat{g}_H \leq 1$, $b_L(\hat{g}_H) \in [0,1]$ and $g_H(\hat{b}_L) \in [0,1]$. Therefore, Brouwer's fixed point theorem implies that a fixed point of $g_H\left(b_L(g_H)\right)$ exists for $g_H \in [0,1]$. This fixed point constitutes an equilibrium, proving existence. Figure A1 plots the reaction functions for two cases, $f = f_1$ and f_2, where $f_2 > (1 - \text{prob}(x_H | y_L)) > f_1$, which implies $T < 0$ for f_1 and $T > 0$ for f_2. The equilibrium strategies are the intersections of the two reaction functions.

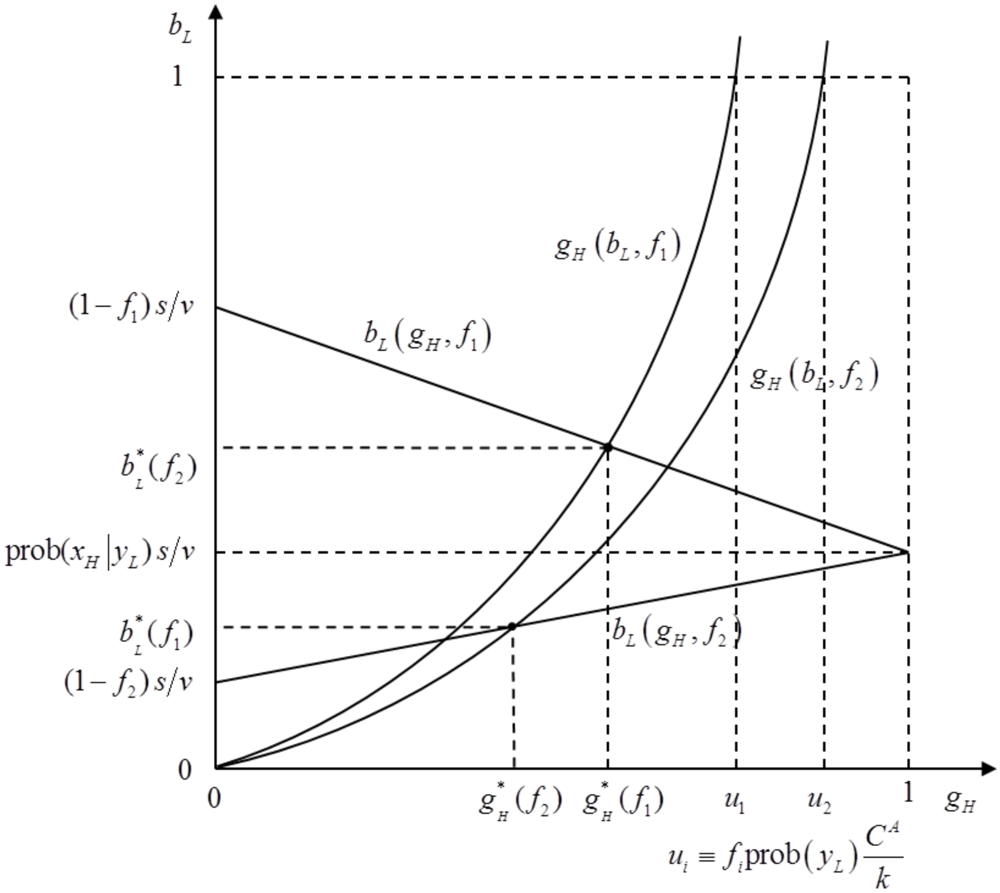

Figure A1: Equilibrium earnings management and audit effort

Uniqueness follows directly from the linearity of b_L and concavity of g_H, which imply that there can be only a single crossing of the two functions over the feasible domains. A special case is $f = 1$. Here, $g_H = \hat{g}_H = b_L = \hat{b}_L = 0$ is a feasible fixed point. A second fixed point could exist if for a g_H around $g_H = 0$, the linear reaction function for b_L is larger than the reaction function for the audit effort. Inverting the first-order condition for g_H gives the value $\bar{b}_L$ for the bias that makes a certain audit effort optimal for the auditor:

$$\bar{b}_L = \frac{\text{prob}(y_H)}{\text{prob}(y_L)} \frac{g_H}{\frac{fC^A}{k} - g_H}$$

In Figure A1, this function is the auditor's reaction function if g_H is assumed the independent variable. A necessary condition for a second fixed point is that

$$\frac{\partial \bar{b}_L\left(g_H=0, f=1\right)}{\partial g_H} < \frac{\partial b_L\left(g_H=0, f=1\right)}{\partial g_H}$$

which implies

$$\frac{\text{prob}\left(y_H\right)}{\text{prob}\left(y_L\right)}\frac{1}{C^A\!/\!k} < \frac{s}{v}\text{prob}\left(x_H\left|y_L\right.\right)$$

$$p\left(1-\alpha\right)+\left(1-p\right)\beta < \frac{s}{v}\frac{C^A}{k}p\alpha$$

$$\underbrace{\left(\frac{1}{\alpha}-1\right)}_{>1\text{ due to }\alpha<\frac{1}{2}} + \underbrace{\frac{\left(1-p\right)}{p}\frac{\beta}{\alpha}}_{>0} < \frac{s}{v}\frac{C^A}{k}$$

The left-hand side of this inequality is greater 1, whereas the right-hand side is less than 1 because v is large and $C^A/k < 1$. Therefore, there does not exist a second equilibrium at $f = 1$ in the feasible domain.

Next, we derive explicit solutions for the equilibrium strategies b_L^* and g_H^*:

$$b_L = \frac{s}{v}\left[(1-f)+\hat{g}_H\left(\text{prob}(x_H\left|y_L\right.)-(1-f)\right)\right]$$

Solving for $\hat{g}_H$ implies $\hat{g}_H = \dfrac{b_L\frac{v}{s}-(1-f)}{\text{prob}(x_H\left|y_L\right.)-(1-f)}$. The optimal g_H given $\hat{b}_L$ is

$$g_H = \frac{1}{1+\dfrac{1-\text{prob}(y_L)}{\text{prob}(y_L)\hat{b}_L}}\frac{fC^A}{k}$$

Equating $g_H = \hat{g}_H$ yields a quadratic equation

$$\underbrace{\frac{v}{s}}_{\equiv T_2}b_L^2 - \underbrace{\left[\frac{fC^A}{k}T - \frac{v}{s}\left(\frac{1-\text{prob}(y_L)}{\text{prob}(y_L)}\right)+(1-f)\right]}_{\equiv T_1}b_L - \underbrace{(1-f)\left(\frac{1-\text{prob}(y_L)}{\text{prob}(y_L)}\right)}_{\equiv T_0} = 0$$

The solution of $T_2 b_L^2 - T_1 b_L - T_0 = 0$ is $b_L = \dfrac{T_1 \pm \sqrt{T_1^2+4T_0T_2}}{2T_2}$. The T_i are (exogenous) constants. $T_0 > 0$ and $T_2 > 0$, implying $\sqrt{T_1^2+4T_0T_2} > \left|T_1\right|$. The sign of T_1 is indeterminate. If T_1

> 0, then the solution for b_L must be the positive root because otherwise b_L < 0, which is not feasible. If T_1 < 0, b_L must also be the positive root for the same reason. Therefore, the equilibrium earnings management is

$$b_L^* = \frac{T_1 + \sqrt{T_1^2 + 4T_0 T_2}}{2T_2}$$

The explicit solution for g_H^* follows from

$$g_H = \frac{\text{prob}(y_L)\hat{b}_L}{\text{prob}(y_L)\hat{b}_L + \text{prob}(y_H)} \frac{fC^A}{k}$$

$$\text{prob}(y_L)\hat{b}_L\left(g_H - \frac{fC^A}{k}\right) + g_H \text{prob}(y_H) = 0$$

Inserting the equilibrium condition $\hat{b}_L = b_L$ yields

$$\text{prob}(y_L)\frac{s}{v}\left[(1-f) + g_H \underbrace{\left(\text{prob}(x_H|y_L) - (1-f)\right)}_{\equiv T}\right]\left(g_H - \frac{fC^A}{k}\right) + g_H \text{prob}(y_H) = 0$$

$$Tg_H^2 + \underbrace{\left(\frac{1-\text{prob}(y_L)}{\text{prob}(y_L)}\frac{v}{s} + 1 - f - \frac{fC^A}{k}T\right)}_{\equiv T_4} g_H - \underbrace{(1-f)f\frac{C^A}{k}}_{\equiv T_3} = 0$$

This is a quadratic equation $Tg_H^2 + T_4 g_H - T_3 = 0$ with solutions $g_H = \dfrac{-T_4 \pm \sqrt{T_4^2 + 4TT_3}}{2T}$. T_3 > 0, and T and T_4 can be positive or negative. Suppose T > 0. Then $\sqrt{T_4^2 + 4TT_3} > |T_4|$ and regardless of the sign of T_4 the positive root is the only solution with g_H > 0. If T < 0, then T_4 > 0 and $\sqrt{T_4^2 + 4TT_3} < T_4$. A solution in real numbers requires that $T_4^2 + 4TT_3 \geq 0$, i.e., $T_4^2 \geq 4T_3|T|$. This must hold because there exists a unique equilibrium (b_L^*, g_H^*) in the feasible range. Denote the two roots $g_H^- > g_H^+$. The reaction function is

$$\bar{b}_L = \frac{\text{prob}(y_H)}{\text{prob}(y_L)}\left(\frac{g_H}{\frac{fC^A}{k} - g_H}\right)$$

This function is a hyperbole that provides positive b_L for small g_H and negative b_L for large g_H. Given this functional form, the positive root g_H^+ is the feasible solution, that is

$$g_H^* = \frac{-T_4 + \sqrt{T_4^2 + 4TT_3}}{2T}$$

Finally, consider the special case $T = 0$, where both numerator and denominator of g_H^* are zero and the quotient is not properly defined. Applying de L'Hospital's rule to g_H^* yields

$$\lim_{T \to 0} g_H^* = \lim_{T \to 0} \frac{\left(\dfrac{4T_3}{2\sqrt{T_4^2 + 4TT_3}} \right)}{2} = \frac{4T_3}{4T_4} = \frac{T_3}{T_4} > 0 \text{ if } T = 0 .$$

The same solution obtains if $Tg_H^2 + T_4 g_H - T_3 = 0$ is solved for g_H at $T = 0$. □

Proof of Corollary 1

We prove first the results for b_L^*. As shown in the proof of Proposition 1, b_L^* is implicitly defined by

$$B \equiv T_2 b_L^2 - T_1 b_L - T_0 = 0$$

where $T_0 = (1-f)\dfrac{\text{prob}(y_H)}{\text{prob}(y_L)}$, $T_1 = \dfrac{fC^A}{k}T - \dfrac{v}{s}\dfrac{\text{prob}(y_H)}{\text{prob}(y_L)} + (1-f)$, $T_2 = \dfrac{v}{s}$, and $T = \text{prob}(x_H \mid y_L) - (1-f)$. To save notation, we drop the asterisk on b_L^*. The total differential with respect to parameters $j = s$, f, and C^A/k is

$$\frac{\partial B}{\partial j} + \frac{\partial B}{\partial b_L}\frac{db_L}{dj} = 0 \Rightarrow \frac{db_L}{dj} = -\left(\frac{\partial B}{\partial j}\right)\left(\frac{\partial B}{\partial b_L}\right)^{-1}$$

where $\dfrac{\partial B}{\partial b_L} = 2T_2 b_L - T_1 = 2T_2\left(\dfrac{T_1 + \sqrt{T_1^2 + 4T_0T_2}}{2T_2}\right) - T_1 = \sqrt{T_1^2 + 4T_0T_2} > 0$. Thus,

$$sign\left(\frac{db_L}{dj}\right) = -sign\left(\frac{\partial B}{\partial j}\right) \text{ for each } j.$$

Part (i): $\dfrac{\partial B}{\partial s} = -\dfrac{v}{s^2}b_L^2 - b_L\dfrac{v}{s^2}\left(\dfrac{\text{prob}(y_H)}{\text{prob}(y_L)}\right) \le 0$, which implies $\dfrac{db_L}{ds} \ge 0$ (strictly for $b_L > 0$).

Part (ii):

$$\frac{\partial B}{\partial f} = b_L\left(-\frac{C^A}{k}T - \frac{fC^A}{k} + 1\right) + \frac{\text{prob}(y_H)}{\text{prob}(y_L)}$$
$$= b_L\left(1 + (1-2f)\frac{C^A}{k}\right) + \frac{1}{\text{prob}(y_L)}\left(\text{prob}(y_H) - b_L\frac{C^A}{k}p\alpha\right)$$
$$= \underbrace{b_L\left(1 + (1-2f)\frac{C^A}{k}\right)}_{>0} + \frac{1}{\text{prob}(y_L)}\underbrace{\left(p\left(1-\alpha\left(1+b_L\frac{C^A}{k}\right)\right) + (1-p)\beta\right)}_{>0} > 0$$

The signs of the terms above follow from $\frac{C^A}{k} < 1$, $f \le 1$ and $\alpha < 0.5$; and using $b_L < 1$ yields $\frac{\partial B}{\partial f} > 0$. This implies $\frac{db_L}{df} < 0$.

Part (iii): $\frac{\partial B}{\partial (C^A/k)} = -fTb_L$, implying $\frac{db_L}{d(C^A/k)} < 0$ if $T < 0$ and $\frac{db_L}{d(C^A/k)} > 0$ if $T > 0$.

Next, we prove the results for g_H^*. As shown in the proof of Proposition 1, g_H^* is implicitly defined by

$$G \equiv Tg_H^2 + T_4 g_H - T_3 = 0$$

with $T_3 = (1-f)f\frac{C^A}{k}$, $T_4 = \frac{\text{prob}(y_H)}{\text{prob}(y_L)}\frac{v}{s} + (1-f) - \frac{fC^A}{k}T$, and $T = \text{prob}(x_H | y_L) - (1-f)$.

To save notation, we drop the asterisk on g_H^*. The total differential with respect to parameters $j = s$, f, and C^A/k is

$$\frac{\partial G}{\partial j} + \frac{\partial G}{\partial g_H}\frac{dg_H}{dj} = 0 \Rightarrow \frac{dg_H}{dj} = -\left(\frac{\partial G}{\partial j}\right)\left(\frac{\partial G}{\partial g_H}\right)^{-1}$$

where $\frac{\partial G}{\partial g_H} = 2Tg_H + T_4 = 2T\left(\frac{-T_4 + \sqrt{T_4^2 + 4TT_3}}{2T}\right) + T_4 = \sqrt{T_4^2 + 4TT_3} > 0$. Therefore, $sign\left(\frac{dg_H}{dj}\right) = -sign\left(\frac{\partial G}{\partial j}\right)$ for each j.

Part (i): $\frac{\partial G}{\partial s} = -g_H\frac{v}{s^2}\frac{\text{prob}(y_H)}{\text{prob}(y_L)} \le 0$ implies $\frac{dg_H}{ds} \ge 0$ $\left(\text{strictly if } g_H > 0\right)$.

Part (ii):

$$\frac{\partial G}{\partial f} = g_H^2 + g_H\left(-1-\frac{C^A}{k}T-\frac{fC^A}{k}\right)-(1-2f)\frac{C^A}{k}$$
$$= g_H^2 - g_H\left(1+\frac{C^A}{k}\mathrm{prob}\left(x_H\,|y_L\right)\right)+(g_H-1)(1-2f)\frac{C^A}{k}$$
$$= \underbrace{g_H(g_H-1)-g_H\frac{C^A}{k}\mathrm{prob}\left(x_H\,|y_L\right)}_{<0}+\underbrace{(g_H-1)}_{<0}(1-2f)\frac{C^A}{k}$$

If $f \leq 1/2$, $\frac{\partial G}{\partial f} < 0$ and $\frac{dg_H}{df} > 0$. If $f > 1/2$, the last term $\underbrace{(g_H-1)}_{<0}\underbrace{(1-2f)}_{<0}\frac{C^A}{k} > 0$, and then the sign of $\frac{\partial G}{\partial f}$ is indeterminate. Note that $\left.\frac{\partial G}{\partial f}\right|_{f=1} = \frac{C^A}{k} > 0$ because at $f = 1$ we have g_H^* = 0. Due to continuity, $\frac{\partial G}{\partial f}$ must be positive in a range of $f < 1$. In particular, there must exist an $f_0 \in (1/2, 1)$ for which $\left.\frac{\partial G}{\partial f}\right|_{f=f_0} = 0$. At this point, g_H^* attains a maximum over f and $\left.\frac{\partial g_H}{\partial f}\right|_{f=f_0} = 0$ as well. This maximum is unique because

$$\left.\frac{d^2 g_H}{df^2}\right|_{f=f_0} = -\left(\frac{\partial^2 G}{\partial f^2}+\frac{\partial^2 G}{\partial f\,\partial g_H}\underbrace{\frac{dg_H}{df}}_{=0\text{ at }f_0}\right)\left(\frac{\partial G}{\partial g_H}\right)^{-1}+\left(\underbrace{\frac{\partial G}{\partial f}}_{=0\text{ at }f_0}\right)\left(\frac{\partial G}{\partial g_H}\right)^{-2}\left(\frac{\partial^2 G}{\partial g_H\partial f}+\frac{\partial^2 G}{\partial g_H^2}\underbrace{\frac{dg_H}{df}}_{=0\text{ at }f_0}\right)$$
$$= -\left(\frac{\partial^2 G}{\partial f^2}\right)\left(\frac{\partial G}{\partial g_H}\right)^{-1} < 0$$

$\frac{\partial^2 G}{\partial f^2} = -2(g_H-1)\frac{C^A}{k} > 0$, which implies $\left.\frac{d^2 g_H}{df^2}\right|_{f=f_0} < 0$. Because this holds for each (local) extremum, f_0 must be the unique maximum; otherwise, there would exist a minimum over the range of f, which is not the case.

Part (iii):

$$\frac{\partial G}{\partial(C^A/k)} = -g_H fT - f(1-f)$$
$$= -g_H f\left(\mathrm{prob}\left(x_H\,|y_L\right)-(1-f)\right)-f(1-f)$$
$$= -g_H f\mathrm{prob}\left(x_H\,|y_L\right)+(1-f)f\underbrace{(g_H-1)}_{<0} < 0$$

which implies $\frac{dg_H}{d(C^A/k)} > 0$. □

Proof of Proposition 2

The proof proceeds by showing that each of the three cost terms in $E[U^O|a_H]$ in equation (8) increases in *s*, which establishes the optimal bonus $s^* = \underline{s}$.

The first term is the expected compensation net of a claw-back

$$\begin{aligned}E[\text{comp}] &= \left(\text{prob}(r_H) - \text{prob}(\text{error})\right)s \\ &= s\left[(1-p)(1-\beta)b_L^*\left(1-g_H^*\right)(1-f) + (1-p)\beta\left(1-g_H^*\right) + p\alpha b_L^*\left(1-f+g_H^* f\right) + p(1-\alpha)\right]\end{aligned}$$

Differentiating with respect to *s* yields

$$\frac{dE[\text{comp}]}{ds} = \underbrace{\left(\text{prob}(r_H) - \text{prob}(\text{error})\right)}_{>0} + s\frac{d\left(\text{prob}(r_H) - \text{prob}(\text{error})\right)}{ds}$$

The first term is strictly positive, and the second term on the RHS is

$$\begin{aligned}&s\frac{d\left(\text{prob}(r_H) - \text{prob}(\text{error})\right)}{ds} \\ &= s\left(\underbrace{(1-p)(1-\beta)(1-f)\frac{d\left(b_L^*\left(1-g_H^*\right)\right)}{ds}}_{>0} - \underbrace{(1-p)\beta\frac{dg_H^*}{ds}}_{>0} + \underbrace{p\alpha\frac{db_L^*}{ds}\left(1-f+fg_H^*\right)}_{>0} + \underbrace{p\alpha b_L\frac{dg_H^*}{ds}f}_{>0}\right)\end{aligned}$$

The signs of the last three terms follow because $\frac{db_L^*}{ds} > 0$ and $\frac{dg_H^*}{ds} > 0$ (see Corollaries 1 (i) and 2 (i)). The sign of the first term follows from the fact that

$$b_L^*\left(1-g_H^*\right) = \frac{\text{prob}(y_H) + \text{prob}(y_L)b_L^*\left(1-f\,C^A\!/k\right)}{\frac{\text{prob}(y_H)}{b_L^*} + \text{prob}(y_L)}$$

depends on *s* only through b_L^*, and $\frac{db_L^*}{ds} > 0$ implies $\frac{d\left(b_L^*\left(1-g_H^*\right)\right)}{ds} > 0$.

Therefore, the only term that negatively enters the derivative is $(1-p)\beta\frac{dg_H^*}{ds}$, and its magnitude depends on β and *p*. The result that $\frac{dE[\text{comp}]}{ds}$ > 0 requires that this term is "small" relative to the sum of the other terms. A "low" β or a high *p* are sufficient that the negative term is small. Moreover, because the other terms in the partial derivative of $\left(\text{prob}(r_H) - \text{prob}(\text{error})\right) > 0$ and the two other cost terms in $E[U^O|a_H]$ also increase in *s* (see below), there are other conditions. An example is a sufficiently high cost of enforcement action C^O (see below).

The second term of $E[U^O|a_H]$ is the audit fee *A*,

$$A=\underbrace{\text{prob}(m_H)}_{=\text{prob}(y_H)+\text{prob}(y_L)b_L^*}\frac{k}{2}g_H^*(2-g_H^*)$$

The total derivative is

$$\frac{dA}{ds}=\text{prob}(y_L)\underbrace{\frac{db_L^*}{ds}}_{>0}\left(\frac{k}{2}g_H^*\left(2-g_H^*\right)\right)+\text{prob}(m_H)k\left(1-g_H^*\right)\underbrace{\frac{dg_H^*}{ds}}_{>0}>0$$

The third term of $E[U^O|a_H]$ is the expected cost of enforcement,

$$\text{prob}(y_L)b_L^*(1-g_H^*)fC^O$$

$\text{prob}(y_L)fC^O\dfrac{d\left(b_L^*(1-g_H^*)\right)}{ds}>0$ follows from the fact that $\dfrac{d\left(b_L^*(1-g_H^*)\right)}{ds}>0$. □

Proof of Proposition 3

Rewriting (10) yields

$$\begin{aligned}
&E[U^M|a_H]-E[U^M|a_L]\\
&=s\left[\text{prob}(y_H)\left(1-g_H^*\,\text{prob}(x_L|y_H)\right)-\text{prob}(y_H|a_L)\left(1-g_H^*\,\text{prob}(x_L|y_H,a_L)\right)\right]\\
&\qquad+\frac{v}{2}\left[\text{prob}(y_L)b_L^{*2}-\text{prob}(y_L|a_L)b_{LL}^{*2}\right]-V\\
&=s(p-q)\left(1-\alpha-\beta(1-g_H^*)\right)-V+\frac{v}{2}\left[\text{prob}(y_L)b_L^{*2}-\text{prob}(y_L|a_L)b_{LL}^{*2}\right]\geq 0
\end{aligned}$$

The minimum bonus $\underline{s}$ is implicitly defined setting this inequality equal to zero:

$$H\equiv\underline{s}(p-q)\left(1-\alpha-\beta(1-g_H^*)\right)-V-\underbrace{\frac{v}{2}\left(\text{prob}(y_L|a_L)b_L^{*2}\left(a_L|\hat{g}_H\right)-\text{prob}(y_L)b_L^{*2}\right)}_{\equiv D}=0$$

(i): $f = 0$. In this case $g_H^*=0$ and $b_L^*=b_{LL}^*=\dfrac{s}{v}$, which yields

$$H\big|_{f=0}=s\left(p-q\right)\left(1-\alpha-\beta\right)-V-\frac{s^2}{2v}\underbrace{\left(\text{prob}(y_L|a_L)-\text{prob}(y_L)\right)}_{=(p-q)(1-\alpha-\beta)}=0$$

which implies $$\underline{s}(f=0)=\frac{V}{(p-q)(1-\alpha-\beta)}+\frac{\underline{s}^2}{2v}$$

and $$\underline{s}(f=0)=v\left(1-\sqrt{1-\frac{2V}{v(p-q)(1-\alpha-\beta)}}\right)$$

because the smaller root is the solution. The equation has a solution in real numbers for $\underline{s}$ if

$$v>\frac{2V}{(p-q)(1-\alpha-\beta)},$$

which is the precise condition for our assumption that v is "large."

To prove that $\underline{s}(f=0)>\frac{V}{(p-q)(1-\alpha-\beta)}\equiv Z=\underline{s}(f=1)$, assume to the contrary that

$$\underline{s}(f=0)=v\left(1-\sqrt{1-2\frac{Z}{v}}\right)<Z$$

which implies

$$\underline{s}(f=0)=v\left(1-\sqrt{1-2\frac{Z}{v}}\right)<Z\Rightarrow 1-\frac{Z}{v}<\sqrt{1-2\frac{Z}{v}}\Rightarrow\left(1-\frac{Z}{v}\right)^2<1-2\frac{Z}{v}\Rightarrow\left(\frac{Z}{v}\right)^2<0$$

which is a contradiction. Furthermore, $D(f=0)=\frac{\underline{s}^2}{2v}(p-q)(1-\alpha-\beta)>0$.

To prove $\left.\frac{d\underline{s}}{df}\right|_{f=0}<0$ apply the implicit function theorem to H:

$$\frac{\partial H}{\partial f}+\frac{\partial H}{\partial s}\frac{d\underline{s}}{df}=0\Rightarrow\frac{d\underline{s}}{df}=-\left(\frac{\partial H}{\partial f}\right)\left(\frac{\partial H}{\partial s}\right)^{-1}$$

$\frac{\partial H}{\partial f}=\underline{s}(p-q)\beta\frac{dg_H^*}{df}-\frac{\partial D}{\partial f}$ and $\left.\frac{dg_H^*}{df}\right|_{f=0}>0$ at $f=0$.

$$\begin{aligned}\left.\frac{\partial D}{\partial f}\right|_{f=0}&=v\left(\text{prob}(y_L|a_L)b_{LL}^*(f=0)\left.\frac{db_{LL}^*}{df}\right|_{f=0}-\text{prob}(y_L)b_L^*(f=0)\left.\frac{db_L^*}{df}\right|_{f=0}\right)\\&=\underline{s}\left(\text{prob}(y_L|a_L)\left.\frac{db_{LL}^*}{df}\right|_{f=0}-\text{prob}(y_L)\left.\frac{db_L^*}{df}\right|_{f=0}\right)\end{aligned}$$

Recall that $b_{LL}^*=\frac{s}{v}[(1-f)+g_H^*\left(\text{prob}(x_H|y_L,a_L)-(1-f)\right)]=$

$b_L^*-\frac{s}{v}g_H^*\underbrace{\left(\text{prob}(x_H|y_L)-\text{prob}(x_H|y_L,a_L)\right)}_{\equiv\Delta>0}$. Thus, $\left.\frac{db_{LL}^*}{df}\right|_{f=0}=\frac{db_L^*}{df}-\frac{dg_H^*}{df}\frac{\underline{s}\Delta}{v}$, and inserting

yields

$$
\begin{aligned}
\left.\frac{\partial D}{\partial f}\right|_{f=0} &= \underline{s}\left(\operatorname{prob}\left(y_L|a_L\right)\left.\frac{db_{LL}^*}{df}\right|_{f=0} - \operatorname{prob}\left(y_L\right)\left.\frac{db_L^*}{df}\right|_{f=0}\right) \\
&= \underbrace{\underline{s}\left.\frac{db_L^*}{df}\right|_{f=0}}_{<0}\underbrace{\left(\operatorname{prob}\left(y_L|a_L\right)-\operatorname{prob}\left(y_L\right)\right)}_{>0} - \operatorname{prob}\left(y_L|a_L\right)\frac{\underline{s}^2\Delta}{v}\underbrace{\left.\frac{dg_H^*}{df}\right|_{f=0}}_{>0} < 0
\end{aligned}
$$

Therefore, $\left.\frac{\partial H}{\partial f}\right|_{f=0} > 0$.

To determine the sign of $\frac{\partial H}{\partial s} = (p-q)(1-\alpha-\beta) + \underline{s}(p-q)\beta\frac{dg_H^*}{ds} - \frac{\partial D}{\partial s}$, recall that $g_H\big|_{f=0}$ = 0, and from Corollary 1 (i) $\left.\frac{dg_H^*}{ds}\right|_{f=0} = 0$ implying

$$
\left.\frac{\partial H}{\partial s}\right|_{f=0} = (p-q)(1-\alpha-\beta) - \left.\frac{\partial D}{\partial s}\right|_{f=0}
$$

$$
\begin{aligned}
\left.\frac{\partial D}{\partial s}\right|_{f=0} &= v\left(\operatorname{prob}\left(y_L|a_L\right)b_{LL}^*(f=0)\left.\frac{db_{LL}^*}{ds}\right|_{f=0} - \operatorname{prob}\left(y_L\right)b_L^*(f=0)\left.\frac{db_L^*}{ds}\right|_{f=0}\right) \\
&= \underline{s}\left(\operatorname{prob}\left(y_L|a_L\right)\left.\frac{db_{LL}^*}{ds}\right|_{f=0} - \operatorname{prob}\left(y_L\right)\left.\frac{db_L^*}{ds}\right|_{f=0}\right)
\end{aligned}
$$

Inserting $\left.\frac{db_{LL}^*}{ds}\right|_{f=0} = \left.\frac{db_L^*}{ds}\right|_{f=0} - \frac{1}{v}\underbrace{g_H^*\big|_{f=0}}_{=0}\Delta - \frac{s}{v}\underbrace{\left.\frac{dg_H^*}{ds}\right|_{f=0}}_{=0}\Delta = \left.\frac{db_L^*}{ds}\right|_{f=0} > 0$ yields

$$
\begin{aligned}
\left.\frac{\partial D}{\partial s}\right|_{f=0} &= \underline{s}\left(\operatorname{prob}\left(y_L|a_L\right)\left.\frac{db_{LL}^*}{ds}\right|_{f=0} - \operatorname{prob}\left(y_L\right)\left.\frac{db_L^*}{ds}\right|_{f=0}\right) \\
&= \underline{s}\left.\frac{db_L^*}{ds}\right|_{f=0}(p-q)(1-\alpha-\beta)
\end{aligned}
$$

Collecting the results,

$$
\left.\frac{\partial H}{\partial s}\right|_{f=0} = (p-q)(1-\alpha-\beta)\left[1-\underline{s}\left.\frac{db_L^*}{ds}\right|_{f=0}\right]
$$

That is, the sign of the term in square brackets determines the sign of the expression. Recall from the proof of Corollary 1 that $\frac{db_L}{ds} = -\left(\frac{\partial B}{\partial s}\right)\left(\frac{\partial B}{\partial b_L}\right)^{-1}$, where

$$
\frac{\partial B}{\partial s} = -\frac{v}{s^2}b_L^2 - b_L\frac{v}{s^2}\left(\frac{\operatorname{prob}(y_H)}{\operatorname{prob}(y_L)}\right)
$$

Using $f = 0$ and $b_L = s/v$ yields

$$\frac{\partial B}{\partial s} = -\frac{1}{v} - \frac{1}{\underline{s}}\left(\frac{\text{prob}(y_H)}{\text{prob}(y_L)}\right) \Rightarrow -\frac{\partial B}{\partial s} = \frac{1}{v} + \frac{1}{\underline{s}}\left(\frac{\text{prob}(y_H)}{\text{prob}(y_L)}\right)$$

Next consider $\frac{\partial B}{\partial b_L} = 2T_2 b_L - T_1$, which at $f = 0$ leads to

$$\left.\frac{\partial B}{\partial b_L}\right|_{f=0} = 2T_2 b_L(f=0) - T_1(f=0) = 2\left(\frac{v}{\underline{s}}\right)\left(\frac{\underline{s}}{v}\right) - \left(1 - \frac{v}{\underline{s}}\frac{\text{prob}(y_H)}{\text{prob}(y_L)}\right) = 1 + \frac{v}{\underline{s}}\frac{\text{prob}(y_H)}{\text{prob}(y_L)}$$

Now it follows

$$\left.\frac{db_L}{ds}\right|_{f=0} = \frac{\left(\frac{1}{v} + \frac{1}{\underline{s}}\frac{\text{prob}(y_H)}{\text{prob}(y_L)}\right)}{\left(1 + \frac{v}{\underline{s}}\frac{\text{prob}(y_H)}{\text{prob}(y_L)}\right)} = \frac{\left(1 + \frac{v}{\underline{s}}\frac{\text{prob}(y_H)}{\text{prob}(y_L)}\right)}{v\left(1 + \frac{v}{\underline{s}}\frac{\text{prob}(y_H)}{\text{prob}(y_L)}\right)} = \frac{1}{v}$$

We show earlier that $\underline{s}(f=0) = v\left(1 - \sqrt{1 - 2\frac{Z}{v}}\right) < v$, hence, $\underline{s}\left.\frac{db_L^*}{ds}\right|_{f=0} = \frac{\underline{s}}{v} < 1$. Taken together, $\left.\frac{\partial H}{\partial s}\right|_{f=0} = (p-q)(1-\alpha-\beta)\left[1 - \underline{s}\left.\frac{db_L^*}{ds}\right|_{f=0}\right] > 0$. This proves $\left.\frac{d\underline{s}}{ds}\right|_{f=0} < 0$.

(ii): $f = 1$. In this case, $g_H^* = 0$ and $b_L^* = b_{LL}^* = 0$, which implies $D = 0$ and

$$\underline{s}(f=1) = \frac{V}{(p-q)(1-\alpha-\beta)}.$$

We have $\left.\frac{\partial H}{\partial f}\right|_{f=1} = \underline{s}(p-q)\beta\left.\frac{dg_H^*}{ds}\right|_{f=1} - \left.\frac{\partial D}{\partial f}\right|_{f=1}$, where $\left.\frac{\partial D}{\partial f}\right|_{f=1} = 0$ (because $b_L^* = b_{LL}^* = 0$), and $\left.\frac{dg_H^*}{df}\right|_{f=1} < 0$. This implies $\left.\frac{\partial H}{\partial f}\right|_{f=1} < 0$ if $\beta > 0$, and $\left.\frac{\partial H}{\partial f}\right|_{f=1} = 0$ if $\beta = 0$. Furthermore,

$$\left.\frac{\partial H}{\partial s}\right|_{f=1} = (p-q)(1-\alpha-\beta) + \underline{s}(p-q)\beta\left.\frac{dg_H^*}{ds}\right|_{f=1} - \left.\frac{\partial D}{\partial s}\right|_{f=1}$$

Since $\left.g_H^*\right|_{f=1} = 0$, it follows from Corollary 1 (i) that $\left.\frac{dg_H^*}{ds}\right|_{f=1} = 0$. We also have $\left.\frac{\partial D}{\partial s}\right|_{f=1} = 0$ due to $b_L^*(f=1) = b_{LL}^*(f=1) = 0$. This yields $\left.\frac{\partial H}{\partial s}\right|_{f=1} = (p-q)(1-\alpha-\beta) > 0$.

Collecting terms yields

$$\left.\frac{d\underline{s}}{df}\right|_{f=1} = -\left(\left.\frac{\partial H}{\partial f}\right|_{f=1}\right)\left(\left.\frac{\partial H}{\partial s}\right|_{f=1}\right)^{-1} \geq 0$$

with strict inequality if $\beta > 0$. This proves that $\underline{s}$ increases in f at $f = 1$. Due to continuity $\underline{s}$ increases if f approaches $f = 1$ from below.

(iii) The existence of a minimum $\underline{s}(f_1)$ follows immediately from statements (i) and (ii) and continuity. □