Public Development Banks and Credit Market Imperfections*

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Abstract

This paper analyzes the role of a Public Development Bank when banks use a costly screening technology to make credit decisions. We explore three issues: 1) what is the main financial market imperfection to be addressed by the PDB; 2) which types of firms should be optimally targeted by public financial support; and 3) what type of mechanism should be implemented in order to efficiently support the targeted firms' access to credit. We show that, in the presence of costly screening, there will be underprovision of credit, resulting from the inability of banks to appropriate the full benefits of projects they finance. This implies that the misallocation of credit is more pronounced for high value projects. This central result, and its implication that PDBs could play a central role in the financing of high value projects, contrast with the usual emphasis on credit underprovision for relatively weak projects/firms (SMEs, young firms, those without collateral, etc.). We show that a public development bank may alleviate these inefficiencies by lending to commercial banks at subsidized rates or providing credit guarantees, targeting the firms that generate high added value. Though in "normal times" PDB lending and credit guarantees are shown to be equivalent, lending is preferred when banks are facing a liquidity shortage, while when banks are undercapitalized, a credit guarantees program is best suited to alleviate the constraints banks' face. Direct lending by the PDB to the targeted industries could

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be superior to these subsidies to private lending, but only if the PDBs corporate governance is strong.

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1 Introduction

The financing of businesses by specialized public institutions is a pervasive feature of financial markets, whether in less developed economies, emerging or developed ones. Regional and global associations of Public Development Banks (PDBs) have over 280 members around the world, some of them large players in the credit markets of their respective countries.[1] The activities of these institutions are varied both in scope and focus. Some of them offer financing to a broad base of clients, while many others target particular types of firms, such as Small and Medium Enterprises, startups, or nascent sectors (Figure 1). They also differ in the way they intervene: while some lend directly to businesses, others offer loans that are intermediated by private financial institutions (Figure 2). Many–73%, according to the Global Survey of Development Banks–offer public guarantees instead of, or in addition to, providing credit.

Despite the pervasiveness of PDBs and the diversity of targets and models through which they intervene in the financial market, it is not clear which particular financial frictions PDBs should alleviate and which instrument, among those used by these institutions, is best suited for dealing with those frictions. Literature and practice have focused on financial market imperfections that imply credit underprovision for relatively weak projects/firms. Most PDBs, for instance, emphasize lending to SMEs (e.g. Figure 1). Theory has had a similar focus: PDB activity has been studied as a solution for the underprovision of credit for projects with negative low present value but positive externalities (Hainz and Hakenes, 2012); or for firms rationed out of credit because of moral hazard (Arping et al., 2010).[2]

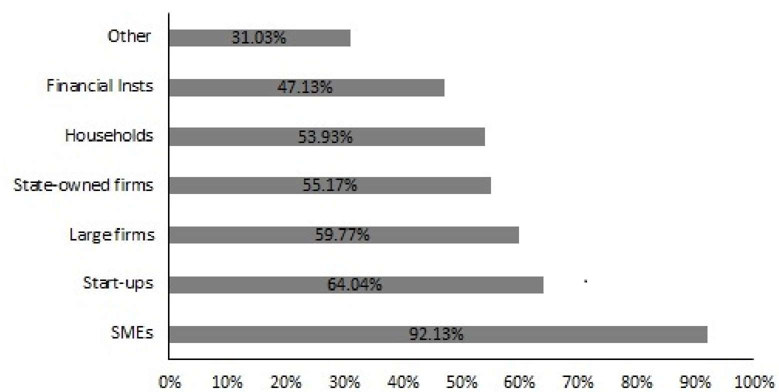
In this paper, we study the role of a public development bank in the context of a model where banks use a costly screening technology to make credit decisions, and where they face at least some competition. The implication is credit underprovision resulting from the inability of banks to appropriate the full benefits of projects. High value projects are rationed out of credit because of this reason, leading to an inefficient allocation of resources to lending. This central result, and its implication that PDBs could play a central role in the financing of high value projects, contrast with the usual emphasis on relatively weak projects.

While there is no single definition of a Public Development Bank, for the sake of precision, we follow the UN (2009), which defines Public Development Banks

[1] Respondents of the World Bank´s Global Survey of Development Banks report participations in assets of between 9% and 19% in the respective market (Luna-Martínez and Vicente, 2012). Lazzarini et al (2014) report that the Brazilian Public Development Bank, BNDES, represents over 20% of loans in the Brazilian credit market, and amount to almost 10% of GDP.

[2] The theoretical literature on banking also provides a number of models where relatively weak firms will not have access to funding in spite of the fact that the project they want to finance has a positive net present value. This is the case of firms with a limited credit history (Diamond, 1991), lack of collateral (Holmstrom and Tirole, 1997, Ruckes, 2004) or, simply, risky (Bolton and Freixas, 2000). PDBs may play a role in alleviating financial imperfections in all of these contexts.

Figure 1: Fraction of PDB's that lend to:



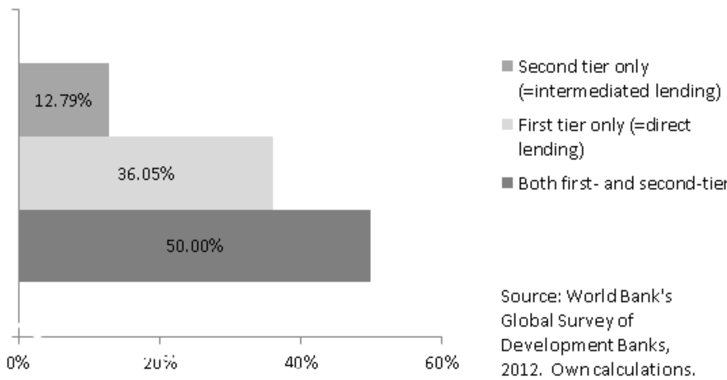
as *"financial institutions set up (by the government) to foster economic development, often taking into account objectives of social development"*.[3] Within this context we focus, in particular, on these institutions' activities regarding the provision of access to funding for businesses.

Our model elaborates on standard costly information extraction, a major building block that has been developed to justify the existence of banks and their role in financial markets. It seems natural, in order to study the role of PDBs, to focus on a financial market imperfection that exists at the bank level rather than at the level of firms–such as moral hazard on the side of firms, and externalities on other businesses (Arping et al, 2010; Heinz and Hakenes, 2012)–. That is our model focuses on banks' supply of credit, while previous justifications of PDBs activity were based on the limitations of a(solvent) demand for loans. We use this theoretical framework to answer questions such as: 1) what types of firms, if any, should be the target of particular public support programs?; 2) should the public finance of firms take the form of direct or indirect lending? 3) if it takes the form of indirect lending, should the PDB lend to private banks at subsidized rates, or rather provide public guarantees (i.e. loss sharing)?

The model considers firms that require funding in order to implement their projects. Firms belong to "industries", which are characterized by a risk profile, so that "industries" may correspond to sectors or types of firms (young, SMEs,...). Within industries, firms can be good or bad, and only good firms have positive net present value projects. This set up has the advantage of allowing us to explore the interplay between our main mechanism and other justifications of public development banks (the targeting of certain sectors, firm sizes and firm ages) and, thus, allows for an overall perspective on the main issues at stake.

[3] UN's (2009) report "Rethinking the Role of National Development Banks". The report goes on to state that these banks often work "*mainly by providing long-term financing to, or facilitating the financing of, projects generating positive externalities*"

Figure 2: Fraction of PDBs that report that they lend...



Source: World Bank's Global Survey of Development Banks, 2012. Own calculations.

The type of a firm is not directly observable to either banks or the government. Still, commercial banks have access to a costly screening technology that yields a signal that may or may not be informative (Ruckes 2004). For any given firm, the bank and the firm will share the project's net present value. In equilibrium some good firms will be credit rationed, so that there is room for public intervention. The reason for the underprovision of screening is that banks do not take into account the externality they create when facilitating firms' access to credit and the rents they generate. The underprovision is more severe for types of firms for which the rents the bank cannot appropriate are larger. The focus of our research is in evaluating the potential effectiveness of different instruments available to a PDB to deal with this credit market failure and to identify the types of firms that should be targeted.

In the process of rigorously stating the issues at hand we will be confronted with the need to define what is specific of a PDB that makes it different, on the one hand, from a commercial bank and, on the other hand from another government agency. We model the PDB as a bank, to the extent that it has access to the same screening technology and the same set of information that other banks have. Still, to the extent that it is publicly owned, it may at the same time pursue a social welfare maximization objective, although tainted by unrelated political objectives.

We evaluate welfare, measured by net expected output, under alternative mechanisms of public financial support, considering the effects of each mechanism on banks' behavior, as well as the implied costs of intervening. We derive the optimal conditions for subsidies to credit, as well as for public guarantees and direct PDB lending, and compare the relative merits of the different arrangements. Though direct PDB lending may reduce distortions from taxation, indirect lending through the financial sector may turn out to be superior given potential political capture and failures in the corporate governance of the public development bank emphasized by previous literature. Within the indirect lending context, in turn, the optimal intervention depends on the returns and risk profiles of the projects that can benefit from the public policy. This has implications for the optimal targeting of government programs, including whether SMEs or other usual suspects are the best possible target.

The theoretical literature on banking also provides models in which credit rationing may arise as a consequence of moral hazard (Holmstrom and Tirole, 1997), or because of liquidity constraints in the financial market (Armendáriz, 1999). We extend our framework to consider these and other related issues and to shed light on how our core financial inefficiency interacts with others in shaping the optimal intervention for a PDB. In particular, we develop extensions to consider moral hazard, bank competition, use of collateral, liquidity and solvency restrictions, and business cycles. As a result, our paper also sheds light on the potential role of PDBs in these different contexts.

The empirical literature has shown that financing constraints affect more starkly particular types of firms. For instance, SMEs report higher financing obstacles than large firms, and the effect of these financing constraints is stronger for them compared to more established firms (See Beck et al. (2008), Beck et al. (2005); Beck et al.,(2006) and Beck and Demirguc-Kunt, 2006 for an overview). Nevertheless, there is also heated debate about whether the more intense obstacles to growth SMEs seem to face indeed make them the optimal target of specific policies. Our framework will contribute to this discussion, and related ones, by identifying features of firms that make them optimal target of policies aimed at alleviating credit rationing.

In the next section we will describe our model and the financial market imperfection it implies, which allows us to discuss the potential role that the PDB could play. We do so by using direct lending by the PDB as a benchmark case. Section 3 will be devoted to a second best policy of indirect intervention by the PDB, via subsidization of credit, which, as we will show can be implemented either via a subsidized lending program or through credit guarantees. While the analysis in sections 2 and 3 focuses on the partial equilibrium, section 4 will consider the impact of competition among banks via endogenous interest rates. Section 5 extends the analysis to explore the role of collateral, liquidity shortages, banks' capital shortages, and business cycles. Section 6 is devoted to the robustness of our qualitative results. Namely, we allow for more flexible specifications for the screening technology and/or the type of moral hazard present. Section 7 concludes by discussing additional implications of our model and suggesting avenues for future research on the role of PDBs.

2 The model

Consider an economy where all agents are risk neutral. Interest rates are normalized to zero. Different industries are characterized by risk parameters p, where p captures the potential probability of success of projects in the industry. Within industries, there are two types of firms, good and bad, in proportions μ and $1-\mu$. Good firms are at the industry's potential, facing probability of success p with an implied positive net present value, while bad firms have a lower probability of success p_{-}, yielding negative net present value. If successful, a project undertaken by a good firm yields an outcome of y per unit of investment, with constant returns to scale up to its full size I, so that a successful project of size I yields yI, while a null return is obtained if the project is unsuccessful ($yp > 1$ while $yp_{-} < 1$).[4] It is easy to generalize our results so as to reinterpret y as the social return rather than the private outcome, to take into account the possibility of externalities.

In order to fund their projects, firms approach banks that have a screening technology. The type (good or bad) of a firm is not observable to banks or the government. The value of p, by contrast, is observed by banks and government. A bank's role in the economy is to screen firms, and thus to weed out bad firms. We initially assume that banks' capital is not a constraint on their credit activity and address the solvency issues and the countercyclical role of PDBs, as well a moral hazard for firms, in extensions to the model.

2.1 Inefficiency in the market for credit

For every industry/risk p, by paying a sunk cost $C(q)$, banks obtain a perfect signal on the firm's type, good or bad (p or p_{-}), with probability q while, with probability $1 - q$, they obtain no signal. We assume $C(q)$ satisfies $C'(q) > 0, C''(q) > 0, C(0) = 0$ and $C'(0) = 0$. If the bank receives a signal it will lend to good firms and deny credit to bad ones. If the bank does not receive a signal, we will assume it does not grant a loan, which occurs when μ is low (namely when $[\mu p + (1 - \mu)p_{-}]\, y < 1$).[5] We assume that screening costs are independent of the firm's project size. We justify our assumption because we associate an increase in size to higher complexity in the structure (balance sheet, multiple business lines,...), but this is compensated by a higher transparency. Because we assume the marginal screening cost is here relevant, we view our framework as focusing on relationship lending, as lending based on credit scoring techniques is probably better characterized by a zero marginal screening cost.

The loan repayment per unit is $R(p)(\leq y)$, and depends upon the structure of competition in the credit market. We take $R\,(p)$ as given for the time being; a later section analyzes the case of competition and the endogenous interest rate $R\,(p)$ and its implications for our central problem. We assume that all banks share the same technology, so that each of them obtains either the same signal or no signal. At this stage, we also assume that there is no collateral, an extension we consider later on.

[4] We do not rule out potential correlations between p, μ and y. To keep the exposition simple, however, our notation does not explicitly recognize these potential correlations.

[5] Notice that, under these assumptions, a bad firm never gets credit. If a bad firm perfectly anticipates this, and applying for loans were costly, that firm would not apply for a loan to start with. In that case our model could be seen as a reduced form for a more complicated formulation with imperfect signals, or firms that do not fully know their type. In the model as it is, however, there is no cost for firms to apply for loans, so in equilibrium we assume all of them do.

With the above assumptions, a bank paying screening cost $C(q)$ receives a positive signal with probability $\mu q(p)$. In that event, it lends an amount I and expects profits $pR(p)-1$ per dollar lent. Banks maximize their profits, choosing a level of screening for every type of risk (industry) p :

$$\max_{q(p)} \mu q(p)(pR(p)-1)I - C(q(p))$$

This is a concave maximization problem with the following first order condition[6]

$$\begin{aligned} \mu\left(pR(p)-1\right)I &= C'(q(p)) \text{ for an interior solution} \\ \mu\left(pR(p)-1\right)I &> C'(1) \text{ for corner solution } q=1 \end{aligned} \tag{1}$$

The case $\mu\left(pR(p)-1\right)I < C'(0)$ for corner solution $q=0$ is excluded as we assumed $C'(0)=0$

The screening level is thus increasing in the banks' return $pR(p)$. In order to identify the financial market imperfections, it is useful to compare the market and the efficient allocation of credit. In doing so, we show that, in equilibrium, banks underprovide screening, with a consequent underprovision of credit in comparison with the efficient allocation level.

The efficient solution, in the perfect information case, results from the maximization of the aggregate output net of the production cost, where the central planner aggregates over industries (risk profiles):

$$\max_{q(p)} \int_0^1 \left[\mu q(p)(py-1)I - C(q(p))\right] dF(p)$$

The solution to this problem implies that the efficient level of screening is characterized by

$$\begin{aligned} \mu\left(py-1\right)I &= C'(q(p)) \text{ for an interior solution} \\ \mu\left(py-1\right)I &> C'(1) \text{ for a corner solution } q=1 \end{aligned} \tag{2}$$

We can now state our first result:

Proposition 1 *If $\mu\left(pR(p)-1\right)I < C'(1)$ market equilibrium leads to underprovision of screening by banks. The size of the inefficiency grows with $p(y-R(p))$, the rent the bank cannot appropriate.*
Proof. *Convexity of $C()$ and the feasibility condition $R(p) \leq y$ yield the result, by direct comparison of (1) and (2), except in the case where $\mu\left(pR(p)-1\right)I > C'(1)$, as $q=1$.* ■

[6] The convexity of $C(q)$ jointly with $C(0)=0$ allow us to dispense with the banks participation constraint, $\mu q(pR(p)-1) \geq C(q)$. At the optimal point this constraint will always be satisfied.

The intuition behind Proposition 1 is simply that screening generates an externality: the good firm that is screened and obtains funding creates an additional output $y - R(p)$ (plus an additional social benefit if y generates an externality) with probability p, an expected profit the bank ignores because, for given p, competition prevents it from charging higher rates to projects with higher value. So, the discrepancy between the market and the efficient provision of screening is precisely given by $\mu p(y - R(p))I$, the benefits that the bank does not fully internalize.

Whether this inefficiency can be partially dealt with, and how, depends upon the instruments available to the PDB. Importantly, the presence of the externality behind this core inefficiency is not due to the use of debt as the banks' financial instrument. Other types of contracts, in particular equity or a combination of debt and guarantees, would generate the same qualitative effect, as the bank's screening incentive to lend would be related to the fraction of the firm's net expected profit the bank appropriates. This fraction is less than one except in the extreme case where the bank buys the project, in which case its role shifts from financier to entrepreneur. Notice that the point is quite general: it applies to any situation where the bank does not appropriate the full net present value of a successful project, so that sophisticated negotiation mechanisms between bank and firm should not eliminate the externality.

We now study some such instruments. As a benchmark, we first examine the problem of a PDB that directly lends to firms. Later, we solve the second best problem where the government, because of asymmetric information, moral hazard or imperfect corporate governance, cannot efficiently lend directly to firms, but is able to act as a principal and design mechanisms to support access to credit by subsidizing banks' and/or firms' activities.

2.2 The Direct Lending benchmark

The most straightforward way to channel credit to those firms that are credit rationed is to structure the PDB as a financial institution, with access to funding and equipped with the same screening technology other commercial banks have. This is a scenario where the PDB will lend directly to firms, but will depart from other banks in that it may pursue the maximization of social net output rather than the maximization of bank benefits, and also potentially be subject to political pressures and have a less efficient governance structure. Conventional wisdom is that the PDB could also (or alternatively) face higher screening costs than other banks.[7]

The simplest way to model the cost of intervention is to introduce a fixed distortion due to taxation. This is to be interpreted as the marginal cost of raising taxes when the tax scheme is optimal. Alternatively, the same parameter may reflect the shadow cost of the PDB budgetary restriction . Denote by λ the distortion associated to the raising of taxes to pay for the costs of government activities.

[7] Though we do not model this possibility directly, our approach could be easily extended so as to capture different sources of inefficiency of direct government lending.

Departure of a PDB from maximizing net output minus the cost of subsidies is to be considered because of its potential lack of independence from politicians, because of imperfect corporate governance, limits to the remuneration policy and other characteristics of many public banks that may lead them to deviate from their intended role in improving credit allocation. Such deviations must not be ignored, as an abundant body of empirical evidence points at cases where credit allocation by PDBs indeed seems to follow political considerations rather than seeking to maximize efficiency. Direct lending by PDBs has been found to increase in election years, and to be targeted to politically valuable costumers or regions, especially in election years (Carvalho, 2014; Cole, 2009; Dinc, 2005; Khwaje and Mian, 2005; Lazzarini et al, 2014; Sapienza, 2004).

We take into account the possibility that the PDB's agenda departs from strict welfare maximization by assuming that, instead of maximizing the net surplus $(py - 1)$, it maximizes a biased objective function $(p(y + \chi(p)) - 1)$. The generality of this formulation has the benefit of being open to a number of interpretations. Indeed, $\chi(p)$ (that we assume could be also be negative) may be interpreted as capturing measurement errors, institutional weakness, corruption, opportunistic behavior by politicians seeking election, or other forms of political capture. It does not capture positive externalities, as these are taken into account in y. In what follows, we refer to the $\chi(p)$ bias as the "political economy drift", with the acknowledgement that alternative interpretations might fit better some environments than others.[8] In addition, it may also be the case that the PDB faces a different (likely higher) screening cost, $\Delta C(q)$.

Assume the total return, including political rents, is such that $(\mu p + (1-\mu)p_-)(y + \chi(p)) < 1$, so that without screening there would be no credit.

The PDB will then maximize:

$$\begin{aligned} &\max_{q(p)} \int_0^1 \{\mu q(p) \left[p(y + \chi(p)) - 1\right] I - \\ &- \left[(C + \Delta C(q(p))(1+\lambda) - \lambda \mu q(p) I (pR(p) - 1)\right]\} f(p) dp \\ \text{s.t.} \quad & 1 \geq q(p); \end{aligned}$$

A simple way to see the objective function would be to provide the PDB with a mandate to maximize $\mu q(p) \left[p(y + \chi(p)) - 1\right] I$ and then to cover the cost $C + \Delta C(q(p))$ out of public funds that cost $1 + \lambda$ per dollar. At the same time, the profit on the loan the firm pays to the PDB is an income to the Treasury and thus has a social benefit of $\lambda \mu q(p) I (pR(p) - 1)$.

Denoting by $\delta(p)$ the Lagrangian multiplier associated with $1 \geq q(p)$, the first order condition with respect to $q(p)$ is given by:

[8] In our current formulation, the political rents $\chi(p)$ are lost if the project is not successful. If we redefine $\chi'(p)$, as $\chi'(p) = p\ \chi(p)$, or $\chi'(p) = \mu p\ \chi(p)$ the alternative interpretation, of political rents unrelated to the success of the project, is obtained. Still, the extreme case of subsidies without screening in exchange for potential or actual campaign support is not covered. This formulation does not cover either cases of bribery, where by providing a subsidy to a firm the politician obtains a kickback, implying $\chi(p) = G(S_F(p))$ where G is an increasing function. Instead, the λ cost of any subsidy is accounted for, independently of the political drift $\chi(p)$.

$$\mu I\left[p(y+\chi(p))-1+\lambda(pR(p)-1)\right]-\frac{dC+\Delta C}{dq}(1+\lambda)-\frac{\delta(p)}{f(p)}=0 \qquad (3)$$

Abstracting first from political rents and from ΔC, it is clear that direct lending by the PDB increases screening with respect to the market solution, and subsequently increases lending, bringing the credit allocation closer to the first best solution. In particular, with direct PDB lending, and focusing for simplicity in the case with $q<1$, condition (3) implies that the PDB equilibrium screening level will be characterized by:

$$\mu I(py-1)=\frac{dC+\Delta C}{dq}+\lambda\left[\frac{dC+\Delta C}{dq}-\mu I\left(pR(p)-1\right)\right] \qquad (4)$$

The left hand side of this equation highlights the fact that the PDB fully internalizes the benefits of funding positive net present value projects, while the last term of the right hand side captures the cost of the intervention compared to the first best. If λ were zero, this equilibrium condition would yield the first best level of screening. With $\lambda>0$, however, the equilibrium implies a level of screening lower than the optimal. To see that this is the case, note that the term $[C'(q(p))-\mu I\left(pR(p)-1\right)]$ is positive when evaluated at the first best q, so that a lower level of q is necessary to satisfy condition (4).[9]

With $\Delta C=0$, this equilibrium would clearly imply higher q than the market solution. However, because λ is strictly positive, the first best would not be reached.

Moreover, if the differential screening cost of the PDB (ΔC) is sufficiently high, or in the presence of what we have called political rents, PDB intervention can do more harm than good. Political rents lead to two biases with respect to the optimal policy in the $\chi(p)=0$ benchmark case. Regarding the level of screening, expression (3) states that a positive $\chi(p)$ will lead to an excess of screening $q(p,\chi(p))$ while a negative value for $\chi(p)$ will lead to an underprovision of screening. Moreover, a high enough level of political rents may imply that $(\mu p+(1-\mu)p_{-})\,(y+\chi(p))>1$: that is, even lending to bad firms yields high enough political benefits that make it attractive to the PDB, leading to inefficient lending. Interestingly, whether this holds or not depends on the industry's p. Certain industries or types of firms may yield particularly high political rents to the politician. The implication is an additional source for inefficiency: the credit allocation is distorted towards these politically attractive groups of firms[10]

[9] The first best satisfies $C'(q)=\mu I\,(py-1)>\mu I\,(pR-1)$.

[10] There are a number of protective layers a PDB may implement in order to isolate itself from political pressures (i.e. reduce the political drift variable). Bancomext and The Business Development Bank of Canada, among others, have attempted to implement three key measures that constitute necessary conditions to limit political interference: first, the bank has invested in as good a screening technology as any other bank, which has implications on salaries and incentives that are sometimes not met by Public Development banks; second, the directors of the board have broad experience in business and are independent; third, the bank's mission is to cover the part of the credit market that is too risky for competing commercial banks.

3 Second best

This section starts, in section 3.1, by defining a second best policy that solves the problem of a PDB setting subsidies to credit activity to provide incentives for banks to increase lending. Sections 3.2 and 3.3 then discuss, respectively, the economic interpretation and possible implementation of this second best policy. Sections 3.5 and 3.6 analyze the robustness of the proposed second best policy to introducing moral hazard and additional externalities, which have been previously proposed as motivations for PDB intervention.

3.1 Optimal subsidy structure

An alternative to direct lending by the PDB is public lending intermediated by a private financial institution. The benefit of indirect lending is that it limits the political drift that may be inherent to direct lending. There are several reasons why this is so: 1) Lending occurs only if the banks deem it profitable; 2) Firms are selected by banks, not by the PDB; 3) The lending or credit guarantees programs do not target specific firms but specific characteristics

Intermediated lending may be subsidized, or not, depending on the conditions banks and firms face. We will now assume the government is able to subsidize the credit activity of banks, and determine to what extent and under which conditions it is optimal to set positive subsidies.[11] We will denote by $S_C(p)$ the per dollar loan credit subsidy, so that the total cost of the loan subsidies to industry p will be $\lambda\mu q I S_C(p)$.

We assume the industry characteristics, p, and subsequently y and $R(p)$ are observable. It is thus possible to implement a policy of credit subsidies that is industry (or risk) dependent. As it is obvious, unconditional subsidies will not affect the agents behavior and, consequently, we directly consider subsidies that are related to the granting of a loan.

Consider the problem with λ, being, as before, the distortion associated to the raising of taxes. This approach may overestimate the cost of the subsidies as the profit the bank obtains from the subsidy will, presumably, be subject to taxation.

$$\max_{S_C(p),q(p)} \int_0^1 \left[\mu q(p)(py-1)I - C(q(p)) - \lambda\mu q(p)I(S_C(p))\right] f(p)dp$$

$$\mu(pR(p) + S_C(p) - 1)I - C'(q(p)) \geq 0 \tag{5}$$

$$S_C(p) \geq 0; \qquad 1 \geq q(p);$$

Constraint (5) holds with equality because the government should subsidize only up to the point where the bank is just induced to provide the second best's q.

[11] Of course, subsidizing banks may imply that the tax structure should be rearranged. If so, banks may receive a subsidy on their lending activity while taxed on their profits.

Denote by $\nu(p)$ the Lagrangian multiplier associated to constraint (5), and let $\delta(p)$ be the multiplier associated with $1 \geq q(p)$.

The first order conditions with respect to $S_C(p)$, and $q(p)$ are, respectively:

$$-\lambda q f(p) + \nu(p) \leq 0 \quad (6)$$

$$\mu I \left[py - 1 - \lambda S_C(p)\right] - C'(q(p)) - \frac{\nu(p) C''(q(p)) + \delta(p)}{f(p)} = 0 \quad (7)$$

We now examine the optimal $S_C(p)$ for a sector characterized by p.

To begin with, notice that since there is no inefficiency when the market leads to full screening ($q = 1$), there is no point in subsidizing the bank when this is the case. The proof is straightforward, because for $q = 1$ constraint (5) is not binding, and consequently, $\nu(p) = 0$, but then condition (6) holds with a strict inequality, which implies $S_C = 0$.

Focusing now on the interior solution for S_C, such that $S_C > 0$ and (6) holds with equality, we can replace (6) into (7) to obtain:

$$\mu I \left[py - 1 - \lambda S_C(p)\right] - C'(q(p)) - \lambda q C''(q(p)) - \frac{\delta(p)}{f(p)} = 0$$

Subtracting constraint (5), rearranging and bringing back to the picture the $q(p) = 1$, case we obtain:

$$S_C(p) = \left[p(y - R(p)) - \frac{\lambda q}{\mu I} C''(q(p))\right] \frac{1}{1+\lambda} \text{ if } q(p) < 1 \quad (8)$$

$$S_C(p) < \left[p(y - R(p)) - \frac{\lambda}{\mu I} C''(1)\right] \frac{1}{1+\lambda} \text{ if } q(p) = 1 \quad (9)$$

where $S_C(p)$ and $q(p)$ satisfy (5).

In the interior solution (8), the subsidy $S_C(p)$ compensates for the upside the bank ignores when it takes its screening decision, so it depends upon $y - R(p)$. Derivation of expression (5) with respect to q yields the marginal cost of driving q up via subsidizing the bank, given by $\frac{\partial S_C}{\partial q} = \frac{C''}{\mu I}$. The level of the subsidy is greater the lower is this marginal cost (see (8)), the larger the distortionary cost of taxation λ, and the lower the first best subsidy $p(y - R(p))$. Notice that the second best $q(p)$ will always be lower than the first best because $\lambda > 0$. If there was no distortion associated to the use of fiscal revenue, $\lambda = 0$, then the first best would, obviously, be obtained.[12]

[12] The distortion λ plays a key role later in our discussion of the merits of direct vs. intermediated public funding.

To understand the implications of this result in terms of the industries that should be targeted, notice that condition $S_C(p) > 0$, together with (8) and (9), imply

$$p(y - R(p)) \geq \frac{\lambda q}{\mu I} C''(q(p)) \tag{10}$$

That is, subsidies should be granted to banks on loans directed to p profiles for which the externality is stronger and the cost of subsidizing is smaller, where the latter happens for larger projects and when the second best q is sufficiently high. It is also the case that subsidizing banks is optimal when the probability of finding a good project is sufficiently high and the cost of taxation sufficiently low.

The above results are collected in the following proposition:

Proposition 2 *The second best efficient solution requires to set a subsidy to bank lending that is increasing in the externality associated with banks screening, decreasing in the distortions associated with using fiscal resources, λ, increasing in μ and p, and decreasing in $C''(q(p))$. The bank will be subsidized for loans to firms that satisfy $p(y - R(p)) \geq \frac{\lambda q}{\mu I} C''(q(p))$, where $q(p)$ is the solution*[13] *to:* $\mu I \left\{ pR(p) + \left[p(y - R(p)) - \frac{\lambda q}{\mu I} C''(q(p)) \right] \frac{1}{1+\lambda} - 1 \right\}) = C'(q(p))$.

Remark 3 *When the market provides $q = 1$, $\mu I \{pR(p) - 1\}) \geq C'(1)$. Thus, in this case $p(y - R(p)) \geq \frac{\lambda q}{\mu I} C''(q(p))$ would imply*
$\left\{ pR(p) + \left[p(y - R(p)) - \frac{\lambda q}{\mu I} C''(1) \right] \frac{1}{1+\lambda} - 1 \right\}) > \frac{C'(1)}{\mu I}$,
and the firm would not be subsidized. In other words, as intuition suggests, if the market already provides full screening, it is optimal for the PDB to abstain from subsidizing credit

Notice that intervention may optimally bring banks to fully screen in cases where the market solution implies $q < 1$. To fulfill constraint (5) the government need only provide S_C up to the point that satisfies:

$$\mu I(pR(p) + S_C(p) - 1) = C'(1) \tag{11}$$

implying $\mu(pR(p) - 1)I - C'(1) < 0$. That is, the banks would not have reached the $q(p) = 1$ level without the subsidy.

In the previous analysis, the optimal choice of S_C has only been implicitly solved for, as condition (8) defines S_C in terms of endogenous variable $q(p)$. As an example with a closed form solution, suppose that $C(q) = \alpha q$. The market allocation would be:

$$q = 1 \text{ for } pR(p) - 1 > \frac{\alpha}{\mu I} \tag{12}$$

$$q = 0 \text{ for } pR(p) - 1 < \frac{\alpha}{\mu I} \tag{13}$$

[13] We are assuming a solution exists, which is generally the case. There are exceptions, one of which is simply the linear cost function where $q(p)$ will neither appear in the left hand side nor in the right hand side.

If (12) holds, then the government would abstain from subsidizing the bank ($S_C = 0$). But if instead we have a corner market solution without screening (13), the government might offer a subsidy just enough to bring the bank to fully screen:

$$S_C(p) = \frac{\alpha}{\mu I} - pR(p) + 1$$

For this subsidy to be optimal, however, condition (9) should hold, so that the social cost of the subsidy has to be lower or equal to the social benefit, $p(y(p) - R(p))$

$$S_C\left(p\right)\left(1+\lambda\right) \leq p(y(p) - R(p))$$

Replacing $S_C(p)$ we obtain the locus of industries which it is efficient to subsidize:

$$p(y - R(p)) \geq \left\{\frac{\alpha}{\mu I} - \left(pR\left(p\right) - 1\right)\right\}\left(1+\lambda\right) \tag{14}$$

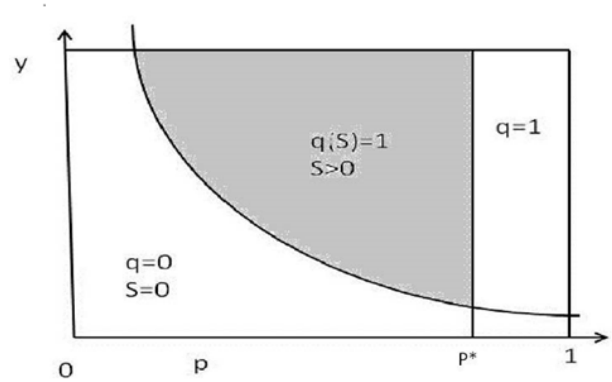
These are industries for which the value of the externality being addressed $p(y - R(p))$ exceeds the (net) cost of screening, adjusted by the distortionary cost of taxing.

Suppose for instance that banks obtain a constant markup m over each loan (so R decreases with p to guarantee that $(pR\left(p\right) - 1)$ is constant). Because in the linear case's market solution banks either screen perfectly or do not screen at all, positive subsidies are efficient only if the market provides $q = 0$. Condition (14) becomes $y \geq \frac{1}{p}\left[\frac{\alpha}{\mu I}\left(1+\lambda\right) - \lambda m + 1\right]$, which makes clear that only high upside value industries (high p, high y) are subsidized, because it is for them that the externality to be addressed is largest, with a constant subsidy $S_C = \frac{\alpha}{\mu I} - m$. The example shows that, although the subsidy to every industry would allow perfect screening, it would not be efficient to subsidize across the board, but only for firms with a sufficiently high y.

Continuing our discussion of the linear screening cost case, Figure 3 illustrates the set of targeted industries in terms of y and p. Targeted industries will be the ones for which the market provides $q = 0$, (which implies $p < p^*$) and $y \geq \frac{1}{p}\left[\frac{\alpha}{\mu I}\left(1+\lambda\right) - \lambda m(p) + 1\right]$ and there will be a trade off between the risk of the project and the required level of y. The grey area corresponds to industries that should be targeted, while the white one (with $S = 0$) are those which subsidization would be inefficient, either because the industry is safe enough to be fully screened in absence of the subsidy, or because its expected value is not high enough to justify the intervention.

The quadratic cost function case provides another useful illustration. As-

Figure 3: targeted industries under linear screening costs



suming $C(q) = \frac{1}{2}\beta q^2$, it is also easy to solve for $q(p)$ and $S_C(p)$:

$$\begin{aligned} S_C(p) &= \frac{p(y - R(p)) - \lambda(pR(p) - 1)}{1 + 2\lambda} \\ q(p) &= \frac{\mu I}{\beta}\left[\frac{py - 1 + \lambda(pR(p) - 1)}{1 + 2\lambda}\right] \end{aligned} \quad (15)$$

The parameter constellations for which $S_C(p) > 0$ are illustrated in Figure 4. Consistent with our discussion of the linear case, the optimal subsidy is positive if p is large enough, and the size of the subsidy also increases with p. The range of p's satisfying this condition expands if the outcome per unit y is higher (light grey line rather than black line). In turn, higher distortionary cost of taxation, λ, reduces the range of subsidized p's and the size of the subsidy (dashed line).[14]

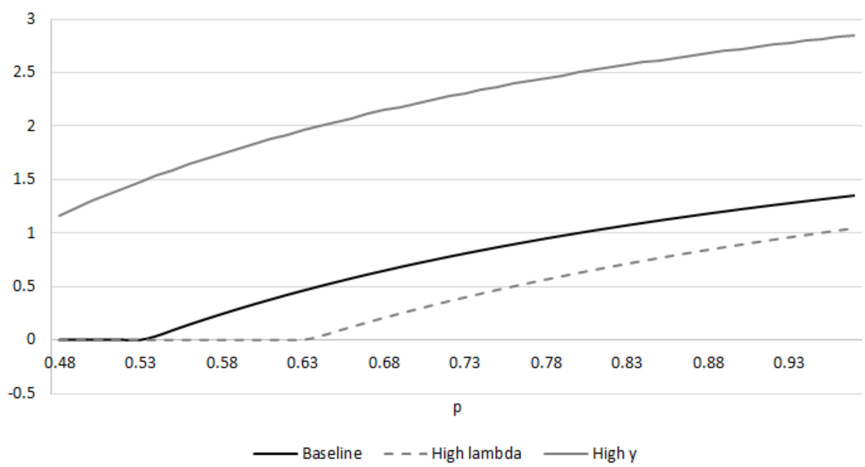
3.2 Economic Interpretation

Our setup highlights the central role of the externality that leads to screening underprovision: financiers do not fully internalize the benefits of lending because they cannot appropriate them (i.e. $y - R(p) > 0$), and thus put less effort than it would be optimal in obtaining a precise signal about a potential costumer. By pinpointing this specific market failure, the analysis makes clear that a subsidy to banks, conditional on their granting a loan, is a natural intervention. The analysis allows us to clarify which types of firms/loans should be targeted.

In particular, condition (10) implies that the subsidies S_C should target industries characterized by $\mu p(y - R(p)) \geq \frac{\lambda q}{I} C''(q(p))$ (condition 10), but only as long as $\mu I(pR(p) - 1) - C'(1) < 0$ (from equation 11) so that $q(p) < 1$ in absence of the subsidy. This implies the targeted firms are characterized by:

[14] To generate this figure, we choose parameter values that satisfy the modelling assumption that $p\mu y > 1$. In particular, we assume $\mu = 0.7$; $y = 3$ in the baseline and $y = 4.5$ in the high y case; and $p \geq 0.48$.

Figure 4: SB under quadratic screening costs



1. Sufficiently high expected firms' unit profits (i.e. high $\mu p\,(y - R\,(p))$) as this reflects the inefficiency of credit rationing the subsidies intend to remedy, proportional to the benefits not internalized by the bank. It is important to notice that our analysis characterizes the second best for every level of y and μ. The whole analysis carries over to any dependence of y and μ on the industry characteristics, p. As a consequence, it is valid for any level of correlation between p and y on the one hand and between p and μ on the other hand.

2. Industries/types of firms for which the markup that banks obtain on loans $(pR(p) - 1)$ is not sufficient to provide incentives for full screening, so that there is credit rationing (i.e. industries where $q < 1$). In particular, fierce bank competition may justify bank subsidies S_C.

3. Though we have so far assumed that screening costs are not industry-specific, if these costs were to vary across industries our analysis would suggest that subsidies should be directed only to industries/clients for which the marginal screening cost is sufficiently high for there to be screening underprovision.

4. Projects with sufficiently large financing needs I. Notice, however, that this is true only to the extent that screening costs are either independent of project size (as specified in the basic model), or at least do not grow sufficiently fast with I. More precisely, if the slope of $C''(q, I)$ with respect to I is greater or equal to 1, then the optimal subsidy is not increasing in the project's size, and may actually decrease with it. See more on this in section *6*.

Some of these implications challenge the conventional wisdom about valid targets for the public financing of enterprises. Credit for firms/projects with high expected returns is frequently deemed unworthy of subsidizing, under the expectation that they will be particularly well served by the market. Our results, however, point out that these projects may be, in fact, the ones where subsidies will be more effective. Low risk (high p) and high y industries are, in consequence, plausible targets of S_C, except in the extreme case where their risk is sufficiently low that the market would grant $q = 1$ without subsidies. Loans to sectors facing particularly dynamic demand growth, or those to firms with risky but high upside value projects, are plausible targets of this policy.

Our results also make it clear that subsidizing loans for large projects/firms may in fact be optimal given the large expected benefits of these loans, if the screening cost does not depend on the project size or if it does not increase with project size at a sufficiently high speed. One may suspect that marginal screening costs actually decrease with project size (for instance because larger projects/firms are required to disclose more information to authorities), in which case targeting large scale projects would be particularly valuable for PDBs. Still, if large firms have a sufficiently low marginal screening cost, they may be subject to perfect screening (corresponding to the case $q = 1$), in which case there is no need for the PDB support.

It is also clear from these results that external positive effects on other firms (other than the one receiving the loan), often deemed as the justification behind the government financing of enterprises, are not a necessary condition for subsidies to be optimal. Even in their absence, the fact that the financier cannot fully internalize the benefits of lending leads to loan underprovision. Of course, when positive externalities over third parties are present, they constitute an additional reason for an intervention that subsidizes loans, a point that is easily born in our model, by simply reinterpreting y as the total social value of the project.

Our results should also provide an answer to the important question regarding whether PDB activities complement or substitute what commercial banks would do in absence of PDB intervention. At least two dimensions are relevant: 1) whether the PDB displaces lending profits from commercial banks to the PDB itself; and 2) whether the subsidies channeled through the PDB simply create rents for banks, without addressing the underprovision of credit.

With regards to the first question, it is clear that credit guarantees or intermediated lending will only complement bank profits. Additional taxes to banks' profits that are not conditional on the bank granting of intermediated loans may be used to counteract this unintended consequence of a policy addressed to increasing lending. When it comes to direct lending, on the other hand, the activity of the PDB directly competes with commercial banks' lending, reducing the profits of commercial banks, except in industries where the market solution already provides $q = 1$, where it is optimal for the PDB to abstain from intervening.

As for the second question on whether subsidies are effective in addressing credit rationing, the PDB's activities put forward by our analysis effectively expand credit to firms with positive net present value, compared to the market solution. This is true both in the direct lending model and the second best solution. While banks' profits do increase in the second best compared to the market solution–as both credit expands and the profit per dollar lent increases– they do it in a manner directed at providing incentives to supply credit, and effective to do so. Consequently, PDB intermediated lending in our model is clearly complementary to private credit activity.

One dimension over which PDB intervention is expected to complement private financing is the provision of long term credit (Armendáriz, 1999; The World Bank, 2016; Smallridge and De Olloqui, 2011). This focus is also supported by our analysis, to the extent that long term credit entails much higher screening costs than short term credit; long term lending implies risks that demand more careful evaluation of the loan. Moreover, projects that have particularly high value will frequently require longer term financing than others (The World Bank, 2016). In our analysis, the PDB precisely focuses on dealing with the underfinancing of projects of particularly high value, a problem that arises in the presence of positive marginal screening costs and that is more acute when these costs are higher.[15]

3.3 Implementation: Subsidized Lending vs. Credit Guarantees Programs

The simplest interpretation for the credit subsidy is as a direct subsidy $S_C(p)$, per dollar of loan, which makes it conditional on the loan being granted. This is not the most usual practice, however. The effects of the subsidy could be reached alternatively by funding credit in conditions that entail an implicit subsidy to the credit activity. Since what is relevant here is to reach a level of $pR(p) + S_C(p) - 1$ that would lead banks to increase their level of screening to the second best level, a policy of subsidized funding to banks at below the market rate, $1 - \delta$, will lead to the same result provided the terms of the loan, $R(p)$, are agreed beforehand[16]. Otherwise cheaper funding might simply be a source of rents for the bank. By setting $\delta = S_C(p)$, the second best allocation will be reached.

Alternatively a policy of credit guarantees will also allow to reach the second best allocation, although with a strong limitation imposed by the banks' incentives to screen. Indeed, too generous a credit guarantee policy would lead the bank to prefer lending to the average firm and save on the screening costs.

A fully subsidized credit guarantees policy will be defined by a payment of an amount $G(p)$ to the bank in case the firm defaults.. In terms of the bank incentives, credit guarantees imply that the bank return on a loan will be $pR(p) + (1 - p)G(p) - 1$. Consequently, the credit subsidy can be imple-

[15] It has also been suggested that one of the reasons why PDBs are particularly useful when coming to long-term financing is their greater ability, compared to private bank, to rely on long term financing (Smallridge and De Olloqui, 2011). In our analysis, this would imply the PDB has a lower cost of funding in this segment of the market which would then decrease the cost to tax payers.

[16] In section 4 we explicitly model competition, so that$R(p)$ becomes endogenous, and examine the impact on the optimal subsidy as the credit subsidy is passed down to firms.

mented by setting $G(p)$ so that $(1-p)G(p) = S_C(p)$, or $G(p) = \frac{S_C(p)}{1-p}$.[17] When the credit guarantee program implies the payment of a fee $F(p)$ by the firm, the expression is unchanged although the cost to the firm will be increased by the amount of the fee. Notice that in our framework there is no difference between a credit guarantees program and selling a credit default swap at the subsidized price.[18]

3.4 Comparing direct government lending vs. second best

In order to compare direct government lending and the second best allocation obtained through subsidies, it is useful to consider, first, the hypothetical case of an unbiased public bank, that is with $\chi = 0$. In this case, it is easy to see that direct lending is superior to the second best allocation.

Remark 4 *When direct lending is unbiased ($\chi = 0$) and the same screening technology is used ($\Delta C(q) = 0$), it is superior to the second best allocation. This is the case because the value of the objective function is higher, as the cost of intervention is reduced because $pR(p) - 1 - C(q(p)) > 0 > -\mu q S_C$ while the direct lending maximization problem feasibility set is larger as the bank incentive constraint is dropped from the program.*

By continuity, the previous remark implies that for small levels of political drift χ, direct lending is preferred, while for larger levels the indirect intervention through the subsidization of banks and firms will be preferred.

Nevertheless, the empirical evidence already mentioned seems to suggest that, at least for some PDBs, the bias χ is quite significant. One possible reason is that the PDB does not have access to the same information about risk profiles p than private banks. Another is institutional weakness leading to the direct lending process not being autonomous with respect to the government and the objectives and constraints of its leaders. Other causes are corruption and more stringent legal constraints that bind public agencies compared to private institutions. In contexts where any of these reasons weigh sufficiently, the distortion that χ brings to PDB lending outweighs its benefits and the implementation of a direct lending program by the PDB will be inefficient.

[17] A bank will prefer to screen firms rather than to lend to the average firm provided:

$$\mu q(p)(pR(p) + (1-p)G(p) - 1)I - C(q(p)) \geq$$
$$\mu(pR(p) + (1-p)G(p) - 1)I + (1-\mu)(p_- R(p) + (1-p_-)G(p) - 1)I$$

We will assume this condition is satisfied, which implies $G(p)$ is lower than the threshold $\overline{G(p)}$, for which the equation holds with equality.

[18] A policy where the PDB charges banks for the credit guarantee is also possible. In such a case, if the PDB charges an amount γ per dollar of loan for the guarantee $G(p)$, the subsidy would be implemented by setting $(1-p)G(p) - \gamma = S_B(p)$, so that charging for the credit guarantee allows to extend their amount.

3.5 Firms' Moral Hazard

Suppose now that firms may engage in moral hazard behavior, as in Holmström and Tirole (1997), which, as we will see, may lead to additional underprovision of credit. This is the focus of Arping et al.'s (2010) model of PDBs. Their assumptions can be easily brought into our model. In particular, firms are able to choose a project that yields private benefits B at the expense of a lower probability of success, $p - \Delta p$ or $p_{-} - \Delta p$. Projects by firms that engage in moral hazard behavior yield a negative expected return, even if the firm is good: $(p - \Delta p)y < 1$ (and a fortiori $(p_{-} - \Delta p)\, y < 1$).

For a given repayment $R(p)$, the firm will choose the high probability of success project, rather than enjoying the private benefits if and only if:

$$p(y - R(p))I \geq (p - \Delta p)(y - R(p))I + B \tag{16}$$

that is,

$$R(p) \leq y - \frac{B}{I\Delta p} \tag{17}$$

In other words, for firms to avoid engaging in moral hazard behavior, banks must leave them a sufficiently high rent. The maximum repayment must be in line with the pledgeable income $y' \equiv y - \frac{B}{I\Delta p}$. For the sake of simplicity, we will assume condition (17) is satisfied for any p larger than some floor level $\underline{p}$, but a different set of assumptions could be made with only minor impact in the formal analysis.

When condition (17) is satisfied, the moral hazard constraint is not binding and the above results of the costly screening problem developed in the previous sections simply applies. Yet, firms with p outside this range will be rationed out of credit for any given level of the repayment. This opens additional room for public intervention.

Consider a modified second best problem where the PDB offers firms a performance premium, so as to provide them incentives to choose the project with the higher probability of success. Denote this performance premium by $P_F(p)$. The PDB problem is now:

$$\begin{aligned} &\max_{S_C(p), P_F(p), q(p), p^*} \int_{p^*}^{1} \left[\mu q(p)(py - 1)I - C(q(p)) - \lambda \mu q(p) I (S_C(p) + p P_F(p))\right] f(p) dp \\ &\mu(pR(p) + S_C(p) - 1)I - C'(q(p)) = 0 \\ &\left[y + P_F(p) - R(p)\right] I \geq \frac{B}{\Delta p} \\ &S_C(p) \geq 0; \qquad P_F(p) \geq 0; \qquad 1 \geq q(p); \end{aligned} \tag{18}$$

Denote by $\gamma(p)$ the Lagrangian multipliers associated to constraint (18), and let $v\,(p)$ and $\delta(p)$ continue to be the multiplier associated, respectively, with the private bank constraint and the positivity constraint for $q(p)$.

The first order conditions with respect to $S_C(p), P_F(p), q(p)$ and p^* are:

$$-\lambda q f(p) + \nu(p) \leq 0$$
$$-\lambda \mu p q(p) f(p) + \gamma(p) \leq 0 \quad (19)$$
$$\mu I \left[py - 1 - \lambda \left(S_C(p) + pP_F(p)\right)\right] - C'(q(p)) - \\ -\frac{\nu(p)C''(q(p)) + \delta(p)}{f(p)} = 0$$
$$\left[\mu q(p^*) I \left[p^* y(p^*) - 1 - \lambda(S_C(p^*) + pS_F(p^*))\right] - C(q(p^*))\right] = 0 \quad (20)$$

Setting $P_F(p) = 0$ is optimal if $p > \underline{p}$, where $\underline{p}$ is the unique solution to $[y - R(p)]\, I(p)\Delta p = B$. The government optimally sets $P_F(p) = 0$ when $p > \underline{p}$, because in this region constraint (17) holds with strict inequality for any level of S_F.

If $p < \underline{p}$, meanwhile, a positive performance premium ($P_F(p) > 0$) opens up as a possibility. $P_F(p) = 0$ would still be optimal, however, in the specific case where $q(\underline{p}) = 0$. In this case, the sector (p) that just fulfills the no moral hazard condition would not be granted credit. Subsidizing sectors with p even marginally below $\underline{p}$ so that they abstain from moral hazard behavior is worthless, as moral hazard is not the reason why they are not granted credit. If the opposite holds, that is if $q(\underline{p}) > 0$, then it is optimal for the government to creative incentives for some firms to behave, if this makes them credit worthy. This is stated in the following Proposition.

Proposition 5 *If $C(q)$ is strictly convex and banks make positive profits in industry $\underline{p}$ ($\underline{p}R(\underline{p}) + S_C(\underline{p}) > 1$), then $p^* < \underline{p}$.*

In words, if $q(\underline{p}) > 0$, then it is optimal for the government to offer performance premia to marginal firms so that they have incentives to choose the positive net present value project, which in turns makes them credit worthy.
Proof. $P_F(\underline{p}) = 0$, as otherwise there would be a strict inequality:

$[y + P_F(p) - R(p)]\, I(p)\Delta p > B$. Plugging $P_F(\underline{p}) = 0$ into the first order condition (20) implies:

$$\mu I \left[\underline{p}y(\underline{p}) - 1 - \lambda S_C(\underline{p})\right] = C'(q(\underline{p})) + \lambda q C''(q(\underline{p}))$$

On the other hand, maximization with respect to p^* implies:

$$\mu I(p^* y(p^*) - 1 - \lambda(S_C(p^*))) = \frac{C(q^*(p^*))}{q^*(p^*)}$$

Because $pR(p) + S_C(p) > 1$ we have $q(\underline{p}) > 0$ and the strict convexity of $C(q)$ implies $C'(q(\underline{p})) + \lambda q C''(q(\underline{p})) > \frac{C(q^*(p^*))}{q^*(p^*)}$. Thus, we have $\underline{p} \neq p^*$, implying $p^* < \underline{p}$. ■

The above proposition states that, as long as $q(\underline{p}) > 0$, there is always a fringe of firms that it is worth targeting with performance premia (a range of $p < \underline{p}$ such that $P_F > 0$). The intuition is simply that a very small premium will allow the firm to be financed (provided, of course $q(p) > 0$, for which a sufficient condition is the existence of positive profits for the bank) and this will bring an increase in both banks and firms' profits.

If the premium P_F is positive, then $\gamma(p) = \lambda\mu pq(p)If(p)$ and this implies the associated constraint is binding, so that

$$P_F(p) = \frac{B}{\Delta p} - y + R(p)$$

The positivity of $P_F(p)$ implies that subsidies go only to firms that would not have been financed otherwise, as their profit $y - R(p)$ would be lower than $\frac{B}{\Delta p}$. That is, $P_F(p) > 0$ only for $p\epsilon(p^*, \underline{p})$

Now, using the respective first order condition, we derive the value for $S_C(p)$

$$\mu(p(y - R(p)) - S_C(p)) - \mu\lambda(S_C(p) + pP_F(p)) - \lambda q\frac{C''(q(p))}{I} - \frac{\delta(p)}{f(p)} = 0$$

implying, for the interior solution:

$$S_C(p) = \left[p(y - R(p)) - \lambda pP_F(p) - \frac{\lambda q}{\mu I}C''(q(p))\right]\frac{1}{1+\lambda}$$

Consequently, there is some trade-off between the two subsidies, as each dollar of additional subsidy to the firm leads to a decrease of $\frac{\mu\lambda}{1+\lambda}$ in the bank's subsidy. The intuition is that, because a subsidy to a firm creates a distortion $\mu\lambda pP_F(p)$, this comes as a reduction in the benefits $p(y - R(p))$ from a banking subsidy.

The analysis in this section shows that premia to ex post successful firms may be an optimal complement to credit subsidies, but only for marginally high risk firms/sectors ($p < \underline{p}$), as a way to reduce the moral hazard incentives that prevent these firms from accessing credit. Notice also that a pure subsidy to a firm, unconditional on the success of the project would have no effect on the moral hazard constraint, as it would be added both to the left and right hand side of condition (16).

The conditional subsidy $P_F(p)$ can be implemented through a reduction of rates, so that the firm net repayment, if successful, is $R(p)- P_F(p)$. This could be a reimbursement to the firm or to the bank which in the latter case is conditional on the bank offering the rate $R(p)- P_F(p)$. Notice that a credit guarantee contract with the firm (rather than with the bank) would have a negative impact on the moral hazard constraint. Indeed, it would increase the attractiveness of the private benefits and low probability of success project, because if the project fails, the firm will still obtain a positive profit.

3.6 Externalities

Hainz and Hakenes (2012) present a model where the role of PDBs is to finance projects that generate externalities on other sectors or projects, even if they have negative present value which would imply a zero market screening level in our model. A simple reinterpretation of the components of our framework can accommodate this possibility. In particular, interpret y as the social returns of the project, by contrast to the private returns to the owner. The size of the market underprovision of credit, given by $\mu(py-1)I$, captures in this case not only the part of the private benefit that is not appropriated by the financing institution, but also the size of the externality not appropriated by either the bank or the project's owner. Hainz and Hakenes' point at this underfinancing of socially beneficial projects as a major justification for the intervention of PDBs. This is consistent with our framework. Our emphasis, however, is on the fact that these external effects need not exist for the PDB to have a role in improving credit allocation, as financial market imperfections imply underfinancing of even positive net present value projects.

4 Competition and Credit Market Equilibrium

For the sake of clarity, we have so far simplified the analysis by assuming an exogenous loan rate, $R(p)$. Needless to say, in a competitive market the equilibrium will be affected by subsidies to lending, and it may be the case that part or all of the subsidy will be passed down to firms. To deal with these concerns, we now study the market equilibrium and its implications for optimal subsidies to loans. We expand the setup to multiple banks that compete in the lending market.

Modeling credit market competition in an imperfect screening framework requires obtaining the optimal interest rate and screening level setting strategies. We follow, for that purpose, the "classical" approach of Broecker(1990) and Ruckes(2004). In this framework, because screening is not costly to firms, they will simultaneously apply to all banks. Each bank will then screen all the firms and make offers to those that are revealed to be good. Because signals convey perfect information they are perfectly correlated across the banks that obtain a signal. Good firms may receive more than one offer and will then choose to borrow from the bank that offers the lowest interest rate. Because of the absence of capacity constrains,undercutting competitors' rates is always profitable, and this leads to the absence of a pure strategy equilibrium, contrary to other approaches (as in Freixas et al., 2007).

Assume N banks are active in the market. The probability of a bank j, $j \neq i$, not granting a loan to a good firm will be the probability of either getting a good signal but setting too high a repayment or getting no signal, which occurs with probability $1-q$. Restricting the analysis to the symmetric equilibrium case, for bank i to be able to grant a loan, it has to be the case that the $N-1$ other banks j either have obtained no signal or are quoting higher interest rates. Thus, the probability of granting a loan at rate R_i to a good firm is $[q(1-F(R_i))+1-q]^{N-1}$, where $F(Ri) = prob(R < Ri)$ characterizes the equilibrium mixed strategy..

Consequently, when quoting R_i, a bank i confronted with $N-1$ competing banks will have an expected revenue equal to:

$$\Pi(R_i) = \mu q I(pR_i - 1)\left[q(1 - F(R_i)) + 1 - q\right]^{N-1} \tag{21}$$

Proposition 6 *In the equilibrium mixed pricing strategy, banks quote repayments R in the range $(\underline{R}, y')$, where $y' = y - \frac{B}{I\Delta p}$ is the pledgeable cash flow, and $\underline{R}$ is given by:*

$$p\underline{R} = 1 + (1-q)^{N-1}(py' - 1)$$

Proof. See appendix A.

Notice that in a purely competitive equilibrium, banks have positive profits $\mu q I(1-q)^{N-1}(py'-1)$ even for the lowest bound $\underline{R}$, provided $q < 1$. The banks' participation constraint is always satisfied because of the convexity of $C(q)$. ■

In equilibrium, banks will quote repayments R in the range $(\underline{R}, y')$, and good firms will choose the best offer, provided they have at least one offer, which occurs with probability $1-(1-q)^N$. The spread of prices depends upon the difference $y' - \underline{R}$, which, itself depends upon p. Firms with $y' < \underline{R}$ will receive no offer as they would have no incentives to choose the right project. Replacing $\underline{R}$ by its value, we observe $y' < \frac{1+(1-q)^{N-1}(py'-1)}{p}$ is equivalent to $y' < \frac{1}{p}$.[19] Not surprisingly, it is risky firms that will be rationed because of moral hazard.

4.1 Equilibrium Screening Level

Given this equilibrium pricing strategy, it is easy to obtain the optimal level of screening in the absence of a subsidy. The bank maximizes

$$\max_{\widehat{q}} \int_{\underline{R}}^{y'} \Pi(R_i)dF(R_i) - C(q)$$
$$\widehat{q} \leq 1$$

But, because $\Pi(R_i) = K = \mu q I(py'-1)\left[1-q\right]^{N-1}$ (with y' equal to the firm's pledgeable income) the problem is simplified and only an interior solution exists, that satisfies

$$\mu I(py'-1)(1-q)^{N-1} = C'(\widehat{q}) \tag{22}$$

where $\widehat{q}$ is the bank's optimal screening level given other banks' screening q. Notice that $q = 1$ will never hold in a symmetric equilibrium.

[19] If $py' < 1 + (1-q)^{N-1}(py'-1)$, then

$$(1-q)^{N-1}(py'-1) > py' - 1$$

But this implies

$$\left[(1-q)^{N-1} - 1\right](py'-1) > 0$$

Because $\left[(1-q)^{N-1} - 1\right] < 0$, the condition is equivalent to $py' - 1 < 0$

Equation (22) implies that the impact of increased competition, due to a larger number of banks N on the symmetric equilibrium ($\widehat{q} = q(N)$) is to decrease $q(N)$, as, for a given q, a larger N implies a lower $(1-q)^{N-1}$. Still, since the measure of firms that are financed is $(1-(1-q)^N)$, the overall effect of an increase in the number of banks may go in either way.[20]

Consider, as an example, the case of linear screening costs, $C(q) = \alpha q$. While in absence of competition screening in this case, if any, is $q = 1$, the market solution with competition implies $(1-q)^{N-1} = \frac{\alpha}{\mu I(py'-1)}$, so that $q < 1$. Since the right hand side does not change with N, any increase in N must be counteracted with a decrease in q to keep the left hand side constant. Moreover, multiplying both sides by $1-q$, we have $(1-q(N))^N = \frac{\alpha}{\mu I(py'-1)}(1-q(N))$. The right hand side increases with N, making it clear that the $(1-q(N))^N$ must also increase with N. Therefore, the fraction of served firms, $(1-(1-q)^N)$, falls with fiercer competition. If, instead of a linear screening cost we consider a function with a very high $C''(q(N-1)$, (or a continuous piecewise linear cost function), the impact of an additional bank on the screening level q will be negligible or zero and the increase in the number of banks will result in an increased access to credit.

Remark 7 *From equation (22) it is easy to derive the impact of changes in the other banks' screening level q on the bank optimal level $\widehat{q}$ and show that banks' screening strategies are strategic substitutes.*

Remark 8 *It is interesting to observe the connection between firms' moral hazard and screening, because at the limit point $y' = \frac{1}{p}$ equation (22) implies the screening level is zero, so that banks will not lend anyway. The linear screening cost example makes this connection clear: the equilibrium screening level is directly related to the pledgeable income.*

4.2 Optimal Subsidy Policy

PDB intervention implies that the mixed pricing strategy of banks becomes a pure strategy. Recall, first, that the probability of a firm being financed in equilibrium is $\mu(1-(1-q)^N)$. Second, if a firm receives a prime P_F, it will satisfy $y' + P_F(p) = \underline{R}$ to make the firm just indifferent between cheating and not cheating. That is, $\underline{R}$ (but not higher interest rates in the mixed strategy's support) will be made feasible by the subsidy to firms.

[20] These counteracting forces emanating from larger competition are reminiscent of Cetorelli and Peretto's (2012) result regarding an intrinsec ambiguity in the effect of bank competition. In their dynamic general equilibrium model with banks that do not screen but provide additional relationship services, the presence of more banks tends to increase the quantity of credit available to firms, but at the same time reduces the benefits of banking services that banks can appropriate, reducing their incentives to provide those services. Our model with costly screening gives rise to a similar result, but precisely because there is screening there is less room for inefficiencies arising from credit misallocation.

The PDB problem may thus now be written:

$$\max_{S_C(p),P_F(p),q(p),p^*} \int_{p^*}^{1} \left\{ \mu I \left[1-(1-q(p))^N\right] [py-1-\lambda(S_C(p)+pP_F(p))] - NC(q(p)) \right\} f(p)dp$$
$$\mu I(1-q)^{N-1}(py'+S_C-1)-C'(q(p))=0 \quad (23)$$
$$p\,(y'+P_F(p))\geq 1$$
$$S_C(p)\geq 0; \qquad P_F(p)\geq 0; \qquad 1\geq q(p);$$

The solution to the PDBs problem, in this pure strategy world, implies that competition undermines the effectiveness of the subsidy to boost credit supply. The reason is that competition forces banks to pass on to the firm, in the form of lower R, part of the subsidy initially intended to make the bank internalize the full benefit of financing the project. Pass-through is larger in the presence of fiercer competition. Paradoxically, a policy of subsidies to credit, $S_C > 0$, may in fact be even more necessary in the presence of fiercer competition, if competition erodes the benefit of financing that the bank internalizes in the market solution. In the previous section, we showed that this is possible. These features of the solution are captured by the following proposition.

Proposition 9 *Optimal credit subsidies to banks and optimal performance premia for firms are given by:*

$$\left\{ S_C^{p>\frac{1}{y'}} = \frac{1}{1+\lambda}\left(p(y-y') - \frac{\lambda C''(q(p))\Upsilon}{\mu I}\right);\ P_F^{p>\frac{1}{y'}} = 0 \right\} \text{ if } p \geq \frac{1}{y'} \quad (24)$$
$$\left\{ S_C^{p<\frac{1}{y'}} = S_C^{p>\frac{1}{y'}} - \frac{\lambda(1-py')}{1+\lambda};\ P_F^{p<\frac{1}{y'}} = (\frac{1}{p}-y') \right\} \text{ if } p < \frac{1}{y'} \quad (25)$$

where $\Upsilon = \frac{\frac{(1-(1-q(p))^N)}{N(1-q(p))^{N-1}}}{(1-q(p))^{N-1}}$.

Proof. See Appendix B ■

Consider first the case $p \geq \frac{1}{y'}$, where moral hazard is not present and, therefore, $P_F = 0$. Condition (24) responds to the usual intuition: subsidy S_B aims at covering the gap between the benefits that the bank appropriates (y' in this case) and those it does not internalize in the absence of the subsidy, but it does not fully close that gap because of the distortionary fiscal cost of the subsidy. Introducing competition, however, further points at the fact that the optimal subsidy also considers the negative effect of fiercer competition on the banks' equilibrium q and on the fraction of the subsidy that the bank does indeed internalize (rather than passing it to the firm), and the counteracting greater probability that the firm encounters at least one positive financier. If, instead, $p < \frac{1}{y'}$ so that $P_F > 0$, the subsidy S_C is adjusted downwards to account for the fact that S_C must be accompanied by a policy premium that imposes additional fiscal costs (captured by $\lambda(1-py')$). In this case, a firm will receive a performance premium $P_F > 0$ when it satisfies $p\epsilon(p^*, \frac{1}{y'})$, but at an interest rate that implies a zero profit for the bank and $q(p) = 0$, so that the subsidy to the firm is ineffective if not accompanied by a credit subsidy.

For further illustration, consider again the example of linear screening costs, $C(q) = \alpha q$, and assume $p > \frac{1}{y'}$, so that $P_F = 0$. Optimal policy implies $S_C = \frac{p(y-y')}{1+\lambda}$ and an increased level of screening with respect to the market solution, given by $(1-q)^{N-1} = \frac{\alpha}{\mu I\left(py' + \frac{p(y-y')}{1+\lambda} - 1\right)}$. The link that competition introduces between screening and moral hazard–highlighted in remark 8 is evident again in the fact that, even with $p > \frac{1}{y'}$, the optimal subsidy depends on pledgeable income y', since the support of equilibrium interest rates depends on y'. The convex costs case, $C = \frac{\beta C^2}{2}$, meanwhile, yields $S_C = \frac{\lambda\beta}{\mu I(1+\lambda)}\left(\frac{1-(1-q)^N}{N^2} - \frac{N^2(1-q)^{N-1}}{N^2}\right)$. The number of banks, N, has an ambiguous effect on S_C: while greater competition reduces the effectiveness of the subsidy to increase banks incentives to screen–because part of the subsidy is passed on to firms via reduced prices–, competition also increases the need for the subsidy.

5 Extensions: collateral, liquidity and capital shortages.

So far, we have considered subsidies to credit and performance premia for firms in a market where firms cannot pledge collateral and banks are able to issue any type of liability and face no constraint, either on their liquidity or on their solvency. When these assumptions are not satisfied, the analysis of optimal subsidy policy changes. To simplify the analysis, we assume away the mixed strategies characteristic of competition and focus solely on the impact of collateral.

5.1 Collateral

To begin with, a preliminary remark on the difference between collateral and PDB credit guarantees is in order. Although in both cases the bank will recover a fraction of the loan in case the borrower defaults, in the collateral case it affects the borrower itself with completely different implications on its incentives to apply for a loan. Because the borrower is not affected by public credit guarantees, their existence will increase the banks' expected return and, therefore, it will also rise the screening level. As mentioned, credit guarantees–when optimal–play the role of a subsidy to lending. Collateral, instead will play a key role in the firms' self selection that may be a substitute for screening as we will now develop.

So far, we have assumed that a bank receiving no signal on a firm will not finance it. Nevertheless, this need not be the case if the firm is to post collateral.[21] In this case, however, it is possible that the amount of the loan the

[21] If property rights do not provide legal certainty to pledging and repossession, however,

firm obtains is constrained by the availability of collateral and the firm's project has to be downsized. We extend now our analysis to the case where agents are endowed with some exogenously given amount of collateral, which we denote V.[22]

As it is standard, we will assume collateral is costly, as the V value of the asset to the bank is lower than its value to the firm, $(1+\delta)V$, where $\delta > 0$. In the present setup, collateral will play two related roles: as a signalling device and in mitigating credit risk.

Signalling allows good firms to separate from bad firms, if the latter are not willing to post collateral. In that sense, the introduction of collateral in the model addresses a more general question about alternative ways for firms to good firms to separate from bad ones.

Let $R_V(p, V)$ be the per dollar repayment on a loan I collateralized with an asset valued V to the bank. Because firms know their types[23], when the value of collateral V is larger than some threshold, only good firms will be ready to pledge their collateral. Define ν_B as the collateral per dollar of loan that leaves the bad firms indifferent between a partially collateralized loan and abstaining from applying for a loan. That is, ν_B satisfies the following condition:

$$p\_(y - R_V(p, V)) - (1 - p\_)(1 + \delta)\nu_B = 0$$

Then, any loan contract with a collateral to loan ratio $\frac{V}{I}$ that satisfies $\nu_B \leq \frac{V}{I}$ will deter bad firms from applying for a loan. Because downsizing has an opportunity cost for the firms, efficient contracts will be characterized by the maximum loan per unit of collateral, that is the minimum $\frac{V}{I}$ that satisfies $\nu_B \leq \frac{V}{I}$. This implies the good firm individual rationality constraint is trivially satisfied, for any contract characterized by a collateral to loan ratio ν_B. This ratio, jointly with V will determine the maximum size I at which the firm will be able to develop its project.

Notice that whenever $\nu_B \leq \frac{V}{I}$ it is unnecessary for banks to screen firms for collateralized lending. The use of collateralized loans implies that all good firms have their projects funded so that there is no credit rationing due to banks' insufficient screening.

Still, depending on the availability of collateral V and on the cost $(1-p)\delta$ of pledging it, the firm may prefer to be screened by the bank. This will be the case if the firm's profits are higher with an uncollateralized loan, that is:

$$p(y - R(p))I^* > p(y - R_V(p))\frac{V}{\nu_B} - (1-p)(1+\delta)V$$

where I^* is the size of the loan required to finance the project without downsizing. The condition is obviously met when collateral is scarce. Still, even

collateral based credit may be quite limited.

[22] This amount will depend, among other factors, upon the legal and institutional features of the economy.

[23] If firms do not know their type, under our assumption of an expected negative present value for the average firm, $(\mu p + (1-\mu)p_-)y < 1$, if banks break even, firms will make losses and, therefore will abstain from asking for a collateralized loan.

if collateral is plentiful, if its cost δ is sufficiently high in comparison to the cost of screening, the condition is also fulfilled[24]. In the following we will assume the condition is satisfied, so that both firms and banks are better off if the bank screens, so that banks' screening and public support to firms are still an issue. Notice, though, that when this condition is not satisfied, and the firm prefers to borrow collateralized because it has sufficient collateral, the policy implication is clear: the PDB should abstain from any intervention.

Because of our interest on studying PDB intervention in the cases where it does have a potential role, we focus on a scenario where, first, banks invest in screening, but if no signal is obtained, they offer the firm the possibility of a smaller collateralized loan that is only attractive to good firms. When this is the case, the objective function of the bank is modified. If the bank obtains a non-informative signal, which occurs with probability $(1-q)$, it will still be able to grant a collateralized loan. The bank profits will now become:

$\max_q \mu \left\{ q(pR(p)-1)I^* + (1-q)\left[(pR_V + (1-p)\nu_B) - 1\right] \frac{V}{\nu_B} \right\} - C(q))$

The first order condition that determines the level of screening will be:

$$\mu \left\{ (pR(p)-1)I^* - \left[(pR_V + (1-p)\nu_B) - 1\right] \frac{V}{\nu_B} \right\} = C'(q))$$

Consequently, the introduction of collateral decreases q through the "spare tire" effect of collateralized lending when the bank obtains no signal. Of course, this does not mean that a policy promoting the use of collateral by protecting creditors' rights to repossession should not be implemented. It simply states that it has a cost in terms of relationship banking and in the lower level of screening it generates. The result is in line with Manove et al.(2001) model of "lazy banks" and has competition policy and regulatory implications. Indeed, on the competition policy side, it implies that the lower the banks' market power in the collateralized market, $pR_V + (1-p)\nu_B - 1$, the higher the level of screening in the uncollateralized segment. On banking regulation, it implies that collateralized loans should have very low capital charge, in line with Basel II and III, and excess of caution will be costly in terms of screening incentives.

Thus, overall, the introduction of collateralized lending will, on the one hand, increase the total output but, on the other hand, diminish the bank's incentive to screen.

Because the subsidy in case of a collateralized loan is not justified, the second best problem becomes:

[24] Because in equilibrium the per dollar expected profits should be equal across banks, we have $pR(p)) - \frac{C(q(p))}{\mu I} = pR_V(p) + (1-p)\nu_B$

A sufficient condition for the above inequality to be satisfied is:

$$\frac{C(q(p))}{\mu I} \leq (1-p)(1+\delta)\nu_B$$

because, in this case, the firm prefers an uncollateralized loan, even in the absence of any downsizing, simply because the expected cost to the firm of losing its collateral is higher than the screening cost to the bank.

Still, this is only an extreme sufficient condition when, in fact, downsizing has an opportunity cost that makes our hypothesis of efficient screening even more natural.

$$
\begin{aligned}
&\max_{S_C(p),P_F(p),q(p),p^*} \int_{p^*}^{1} \{\mu \left[q(p)(py-1)I^* + (1-q(p)) \left(p(y-1)\frac{V}{\nu_B} - (1-p)\delta V \right) \right] \\
&-C(q(p)) - \lambda\mu q I^*(S_C(p) + pP_F(p))\} f(p)dp \\
&\mu \left[(pR(p) + S_C(p) - 1)I^* - [(pR_V + (1-p)\nu_B) - 1] \frac{V}{\nu_B} \right] - C'(q(p)) = 0 \\
&[y + P_F(p) - R(p)]\, I^* \Delta p \geq B \qquad (26) \\
&S_C(p) \geq 0; \qquad P_F(p) \geq 0; \qquad 1 \geq q(p);
\end{aligned}
$$

The main impact of collateral will be on the optimal credit subsidy, as the option of collateralized lending decreases the benefits of screening. When a collateral V could be pledged with the bank, optimal subsidies to credit are given by (see Appendix C for a derivation):

$$
\begin{aligned}
S_C(p) &= S_C^{cll}(p) \text{if } q(p) < 1 \qquad (27) \\
S_C\,(p) &\leq S_C^{cll}(p) \text{ if } q(p) = 1
\end{aligned}
$$

where $S_C^{cll}(p) = \{p(y-R(p)) - \left(p(y-R_V)\frac{1}{\nu_B} - (1-p)(1-\delta) \right) \frac{V}{I^*} - \frac{\lambda q}{\mu I^*} C''(q(p))\} \frac{1}{1+\lambda}$. This expression allows us to identify the industries that should be targeted, that is, such that $S_C(p) > 0$. These industries will be characterized by the following expression:

$$
p(y - R(p)) \geq \left(p(y-R_V)\frac{1}{\nu_B} - (1-p)(1-\delta) \right) \frac{V}{I^*} + \frac{\lambda q}{\mu I} C''(q(p)) \qquad (28)
$$

For a given expected profit, our findings square with the argument that firms lacking the possibility of collateralizing their loans are desirable targets of public financing. In particular, low available collateral V and high minimum required collateral ν_B make it more likely that the above condition is fulfilled. Small and young firms, and those in sectors holding little pledgeable assets (such as services), are likely examples of such targets.

The comparison between the level of the subsidy when there is no collateral, (8) and when there is collateral, (27) shows that the subsidy is much larger in the first case. The explanation is obviously that, in the first case, the social loss of the bank not getting any signal is the loss of $p(y-1)$, while in the second case, the cost is only a lower level of funding for the firm, corresponding to $p(y-1)(I - V(1-\gamma))$,which depends upon the amount of collateral V available at the firm level.

5.2 Liquidity

In our set up, limited access to funds on the side of banks, for instance during financial crises, can be easily modeled through the introduction of an additional constraint limiting the bank's total credit supply in the analysis of the second best[25]. Implicitly, it is assumed that the supply of outside liquidity by monetary policy authorities cannot be altered, and that the PDB is not forced to support an eventual monetary contraction policy (although a reinterpretation of λ could account for the PDB liquidity shortage) and is able to pursue its own lending policy.

In such a framework the banks' choice of screening will take the constraint into account, as they will now maximize

$$\begin{aligned} &\max_{q(p),p^{\grave{u}}} \int_{p^*}^{1} \left\{ \mu q(p)(pR(p) + S_C(p) - 1)I - C(q(p)) \right\} f(p)dp \\ &\int_{p^*}^{1} \mu q(p) I f(p)dp \leq L \end{aligned} \tag{29}$$

The solution to this problem will be, if ϕ is the Lagrangian multiplier associated to the liquidity constraint:

$$\begin{aligned} &\mu(pR(p) + S_C(p) - (1+\phi))I - C'(q(p)) = 0 \\ &\int_{p^*}^{1} \mu C'^{-1}(\mu(pR(p) - (1+\phi))I)) I f(p)dp \leq L \\ &\mu q(p^*)(p^* R(p^*) - (1+\phi))I - C(q(p^*)) = 0 \end{aligned}$$

Not surprisingly the liquidity restriction implies a shadow cost of liquidity that can be interpreted as an interest rate increase.

The PDB will now solve:

$$\begin{aligned} &\max_{S_C(p),P_F(p),q(p),p^*} \int_{p^*}^{1} \left\{ \mu q(p)(py - 1)I - C(q(p)) - \lambda \mu q I (S_C(p) + pP_F(p)) \right\} f(p)dp \\ &\mu(pR(p) + S_C(p) - (1+\phi))I - C'(q(p)) = 0 \\ &\int_{p^*}^{1} \mu C'^{-1}(\mu(pR(p) - (1+\phi))I)) I f(p)dp \leq L \\ &\mu q(p^*)(p^* R(p^*) + S_C(p^*) - (1+\phi))I - C(q(p^*)) = 0 \\ &\left[y + P_F(p) - R(p) \right] I \Delta p \geq B \\ &S_C(p) \geq 0; \qquad P_F(p) \geq 0; \qquad 1 \geq q(p); \end{aligned}$$

Now, depending on the way the subsidies are implemented, they may imply or not additional liquidity for the targeted firms. Under intermediated lending, the PDB will be able to use, in fact two instruments: $S_C(p)$ and $\Delta L(p)$, a credit

[25] This, of course, disregards why and how a liquidity shortage occurs. Considering those reasons would require the modeling of the whole monetary policy framework.

line that will alleviate the liquidity constraint for loans in the industry p and the liquidity constraints.. Let $\phi(p)$ be the Lagrangian multiplier associated to the liquidity constraint (29).

It is easy to prove that, if $\phi > 0$, that is, if the liquidity constraint is binding, the use of $\Delta L(p)$ will always strictly improve upon the exclusive use of subsidies implemented through instruments unrelated to liquidity. Indeed, assume, by way of contradiction, that the optimal structure constrained by $\Delta L(p) = 0$ is obtained. Because in the constraints of the above problem, only the expression $S_C(p) - (1+\phi)I$ appears, it is clear that a positive $S_C(p)$ can be substituted by the equivalent decrease in $\phi(p)$ that is generated by an increase in $\Delta L(p)$. Still, while ϕ is positive, an increase in liquidity will (optimally) generate more lending even if offered to the bank at the market rate rather than at subsidized rate. So, even if the optimal policy may still involve a subsidy, it will be combined with a policy of alleviating the bank's liquidity constraint and thus reduce the opportunity cost of lending for some specific industries p.

5.3 Capital Shortages

The banks' lack of regulatory capital, characteristic of a credit crunch (See Bernanke and Lown, 1991) may also impose a limit to the banks' ability to lend. Although the equation that captures the restriction is similar to the liquidity constraint (29) above, the effects will be quite different. Denote as β the risk weight associated to firms' lending, that is, the coefficient of required capital to extend a given amount of credit. If all the loans to firms have the same risk weight, the constraint will be:

$$\int_{p^*}^{1} \beta\mu q(p) I f(p) dp \leq E$$

If capital shortages are a key constraint, then reducing the loss given default on a loan is a feasible way to soften that constraint. Because a credit guarantees program reduces the banks' risk for the targeted loans as exposure is reduced from I to a fraction $(1 - \frac{G(p)}{I})I$, if $\frac{G(p)}{I}$ is the fraction of losses the PDB commits to cover. PDB credit guarantees program is the right way to alleviate limited credit due to capital constraints. Nevertheless, the impact of credit guarantees will depend upon its regulatory treatment[26] and, in particular upon lifting the need for capital backing in the case of guaranteed loans, as well as (or alternatively) on the credit rating of the PDB. With an ill-rated PDB, credit guarantees by the PDB may not be credible and therefore be ineffective.

[26] The experience of Colombia is particularly illustrative: the number of guarantees provided by the program of public credit guarantees more than doubled after the financial regulator issued a decree (Decree 686, 1999) deeming these guarantees "admisible". Admisibility allowed financial institutions to use the guaranteed amounts towards their capital requirements (Arraiz et al. 2014).

5.4 Business and Credit Cycles

Macroeconomic conditions have a strong impact on credit, with tightening standards associated with lower expected future levels of loans and output (Lown and Morgan, 2001 p.1581). As a result, public financing by PDBs is frequently expected to play a countercyclical role (Luna-Martinez et al., 2012). Our framework provides a rationale for active countercyclical lending by a PDB, as recessions may be times of particularly acute liquidity and capital restrictions for banks, specially when associated with financial crises. Public funding to banks and credit guarantees will help ease liquidity and capital constraints as established in sections 5.2 and 5.3, and may thus be particularly valuable during times of crises. Lending to banks eases starker liquidity constraints associated with the crisis, while credit guarantees help deal with potential increases in the banks' exposures during bad times and the capital shortage it implies. However, the PDB support during a crisis need not incorporate an implicit subsidy. In particular, the public lines of credit that are extended by the PDB to deal with liquidity shortages could charge market rates (section 5.2) and credit guarantees could be issued at their cost, which reduces the cost to taxpayers at a point in time where it could be particularly high.

Crises may also be times when the expected return from new projects is particularly low (low py), reducing, for any given $R(p)$, the externality that implies underprovision of screening, and increasing incentives to engage in moral hazard. Our analysis thus suggests that demand driven decreases in the volume of credit during crises should lead to less active PDB interventions. Bear in mind, however, that our static framework is not well suited to deal with dynamic costs from crises in the presence of credit constraints, and from public lending for working capital.[27] Both fronts may justify active countercyclicality even when recessions are demand driven.

6 Robustness

At this stage it is interesting to examine how robust are our qualitative results. We study how results are affected by different changes in our basic assumptions.

- **Alternative screening technology**

Regarding the screening technology, our framework treats screening as weeding out bad firms. Would the same results hold if, instead we had a screening technology based on an imperfect signal? The answer is affirmative provided screening is costly.

Suppose that screening will provide a signal s on the firms' distribution of cash flows y, generating an ex post distribution with density function $f(y \mid s)$,which is informative about y in the sense of the Monotone Likelihood Ratio

[27] Hallward-Driemeier and Rijkers (2013) and Eslava et al. (2015) estimate that there are long-lasting TFP losses from the inefficient exit of profitable but credit constrained firms. Providing public credit for working capital may help deal with these inefficiencies

Property (MLRP), so that high signals imply a higher probability mass on the high cash flows. When this is the case, the optimal decision for the bank will be to lend whenever the signal is higher than some threshold s^*. The bank choice of screening corresponds then to the precision of the signal s, ranging from a perfect signal $y = s$ at a high cost to no precision at all (in which case $f(y \mid s) = f(y)$) at zero cost. The precision level will result from profit maximization and, again, will not take into account the benefits accruing to the firms of the choice of precision, $p(y - R(p))$.

Still, the analysis of competition will lead to different conclusions, because, signals will not be perfect any longer, so that bad firms will have a chance to be granted credit. This implies, as in Broecker(1990) that when the population of banks increases, the chances of bad firms to obtain credit increases, so that for a given interest rate, the average return on a bank loan may decrease.

- **Screening costs and interest rates that depend on loan size**

We have assumed that the screening cost does not depend upon the size of the project and of the loan. This seems a reasonable yet critical assumption. Indeed, if the screening costs were to be proportional to the projects' size, it would imply that size is irrelevant in the screening decision and small firms would have the same chances of being financed as large firms.

Also, we have assumed $R(p)$ does not depend upon the size of the loan. This implies the bank choice of $q(p)$, when confronted with a repayment $R(p)$, a subsidy, $S_C(p)$, and a size $I(p)$ will result from the first order condition:

$\mu(pR(p) + S_C(p) - 1)I(p) - C'(q(p)) = 0$, with the simplification that it is the marginal cost of screening per dollar of granted loan $\frac{C'(q(p))}{\mu I(p)}$ that matters. Dropping the assumption would simply imply that the bank optimal screening level will result from the total revenue $R(p, I(p))$ and total subsidy $S_C(p, I(p))$. The impact of size will then cease to be linear. The qualitative results would remain the same, with the exception that the need for a subsidy may be starker for project sizes that imply greater screening costs.

- **Industry specific screening costs**

Finally, it is often argued that screening might be more or less costly in different industries (representing, in our framework, groups of firms that share some characteristic).. This is the case, for instance, for SMEs. As stated by Beck et al. (2008, p.1-2)"Both high transaction costs related to relationship lending and the high risk intrinsic to SME lending explain the reluctance of financial institutions to reach out to SMEs". In addition, the scarcity of reliable data on SMEs and the possible manipulation of their financial statements make screening more costly. The same arguments should apply for young firms as well as for young industries. In our model, if repeated lending to the same industry decreases the screening cost, the optimal subsidies should also decrease. When this is the case, subsidies should be directed to "nascent" industries and should disappear from "senescent" industries. If the screening cost is related to relationship lending, then a high turnover in the population of firms makes investing in the relationship less profitable. In our context, this implies considering a screening function $C(q(p), p)$, which is a straightforward extension.

We have considered the screening costs faced by banks and the expected value of projects as exogenous to the banks' activities. However, some public development banks also have consultancy activities directed to both firms and banks. This is the case, for instance, of the Business Development Bank of Canada or Bancoldex (Colombia). In the context of our model this could be associated with the public development bank being able either to increase the distribution of returns or to decrease the cost of screening (e.g. by improving accounting standards, or corporate governance). In both cases this will increase access to credit, and should therefore decrease subsidies for screening to the industry and its cost.

- **Multiple thresholds for firms' Moral Hazard**

Regarding firms' moral hazard, we have assumed a unique solution $\underline{p}$ to the equation $y - R(p) = \frac{B}{\Delta p}$, but the extension to a more general case, even if more cumbersome, is straightforward. It will define N intervals $(p_1, p_2), ...(p_{2N-1}, p_{2N})$ such that for any $p, p \epsilon (p_{2k-1}, p_{2k}), y - R(p) < \frac{B}{\Delta p}$. Then our proof extends and it is possible to show that it is always beneficial to subsidize firms $p_{2k-1} + \varepsilon$ and $p_{2k} - \varepsilon$ for ε sufficiently small, as a very limited subsidy allows $q(p)$ firms in this interval to be financed and generate $\mu q(p)(py - 1)$ additional output which is independent of ε. Other forms of moral hazard, as firm's effort level, could be considered. Appendix C briefly examines an alternative modeling of moral hazard, through the introduction of a cost of effort function at the firm level and the implications it would have, and shows that, again, it would be optimal to subsidize firms with marginally insufficient incentives to exert effort.

7 Conclusion

This paper proposes a framework to analyze the role of PDBs. We argue that the main justification for this type of institution is not to be found in either the positive externalities of some negative net present value projects or on the firms' limits to contract upon their future actions and the moral hazard that those limits imply. Instead, we propose that the main role PDBs may play is to help deal with financial market imperfections. The fact that financial institutions face potentially large screening costs when lending to firms makes the environment we model a natural candidate to analyze the merits of different possible PDB arrangements.

As we show, screening costs imply that some positive net present value firms are suboptimally deprived from funds, thus introducing a major friction in the credit market. Contrary to conventional wisdom this approach points that (inefficient) underfinancing is particularly acute and costly for society in the context of high net present value projects. This conclusion shifts the spotlight from the usual focus on public financing for risky small business that lack sufficient collateral, those with a limited credit history, or those with a low present value. Instead, our results highlight that PDBs should provide incentives for commercial banks to increase their screening of highly profitable firms/projects. When banks are unconstrained, this could be done indifferently through refinancing at low rates or with a credit guarantees program at rates below the market price (CDS). Still, if banks face liquidity constraints, indirect lending dominates, while if they are capital constrained, credit guarantees that reduce the weight of risky assets in the portfolio, will be more effective.

The fact that the credit market failure underlying suboptimal private provision of credit is the presence of screening costs also points to additional components of effective government intervention in the credit market. Of particular importance are efforts to boost innovation in screening technologies, including the strengthening of public sources of information on the productive sector, both at the individual firm level and regarding the economic perspectives of specific industries or types of firms. To the extent that information and screening technologies have public good characteristics, this is a task in which government intervention has a high potential value. In practice, it is not unusual for governments and multilateral to support the banking industry in the strengthening of screening technologies.[28]

Our framework also lends itself naturally to comparing developed and emerging markets in terms of both the potential extent of underfinancing and the potential effectiveness of PDB intervention to successfully address this market imperfection. On the former front, our framework captures some of the reasons why underfinancing may be particularly acute in emerging markets, such as underdeveloped financial sectors with poor screening technologies, poor property rights enforcement, and/or low access to effective collateral. On the latter, it warns that countries with weak state capacity may face particularly high distortionary costs of taxation and also face difficulties in establishing PDBs with a strong enough corporate governance to be able to isolate itself from political pressures and acquire the best banking practices.

One natural question is to what extent our model sheds light on the activities of public financial institutions that provide financing to activities other than productive projects. One particular interesting field is the public financing of real estate. Institutions such as Fannie Mae and Freddy Mac in the United States are also prevalent around the world. While our model ignores issues relevant when discussing public interventions in the mortgage market, such as distributive considerations, it can still shed light on some potential effects of such interventions. In fact, in the context of costly screening of our model, real

[28] For example, the World Bank Group's International Financial Corporation provides advice to financial institutions in assessing the potential of specific types of clients. (See, e.g., IFC's SME Banking Knowledge Guide, 2010, pages 44-48 for an example on advice for screening SMEs). In another front, the Colombian public agency for the financing of innovation runs a program that subsidizes selected bank proposals for the development of screening programs for startups (see https://www.innpulsacolombia.com/sites/default/files/convocatoria_bancos_-a_creer.pdf)..

estate should not be supported by PDBs. This is the case, first, because the loan is likely to be sufficiently collateralized, so that subsidizing credit activity is unnecessary. Second, the marginal screening cost may be zero, as it happens with loans based on credit scoring, so that any subsidy will have no effect on banks incentives to increase the quality of their screening. Consequently, absent considerations regarding redistribution that our model ignores and that might be relevant in the context of subsidized real estate lending, the case for subsidizing residential mortgages is a very weak one. Put differently, the existence of screening costs and imperfect information about buyers is not a plausible reason to argue that private real estate funding is inefficiently low, or that a PDB should provide such funding. Similar arguments have been made by Beck et al (2012) and Sassi and Gasmi (2014).

Assuming screening is costly is clearly a natural view of the banking lending process. Still, alternative financial market imperfections may presumably lead to different conclusions regarding the role of PDBs in improving resource allocation. This as a potentially fruitful area for future research.

Although we cover a number of important extensions, much ground is still to be explored even within the scope of a model based on the existence of screening costs. In particular, the political economy of PDBs, elegantly addressed by Hainz and Hakenes(2012) is condensed in our model to a reduced form parameter. While we consider that the second tier intermediated lending by the PDB is free from political interference, the actual analysis may be much richer, as it is possible to sustain an inefficient senile industry by subsidizing both banks and firms for political reasons. Such deviations from efficiency, in turn, lead to the issue of PDBs corporate governance, an issue that deserves much deeper research, as it is directly linked to the one of government owned firms (and banks).

The analysis of the role of PDBs in the business and credit cycle also goes beyond the excessively streamlined conclusions that our approach yields. The issue of the PDB's access to funds is also to be addressed, and its ability to provide firms with credit over a longer maturity may have an important impact on the ability of firms to undertake long run investments.

To conclude, we believe it is relevant to explore the justification of PDB activity in the light of what is known about financial markets imperfections. The costly screening approach seems natural and provides a simple framework that allows to draw interesting non trivial conclusions. Clearly, as the field has not been widely researched, we expect future contributions to complement our results.

8 References

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9 Appendix

9.1 Appendix A: Proof of proposition 6: pricing strategy under competition

When quoting R_i, a bank i confronted with $N-1$ competing banks will have an expected revenue equal to:

$$\Pi(R_i) = \mu q I(pR_i - 1)\left[q(1 - F(R_i)) + 1 - q\right]^{N-1}$$

Because in a mixed strategy equilibrium all strategies yield the same expected profit, the equality $\Pi(R_i) = K$ allow us to obtain the common cumulative probability distribution $F(R)$, that satisfies $K = \mu q I(pR_i - 1)\left[q(1 - F(R)) + 1 - q\right]^{N-1}$

The repayment R_i is bounded below by the zero profit lower bound,$R_i \geq \frac{1}{p}$ and above by the pledgeable cash flow $y - \frac{B}{I\Delta p}$ that we denote by y'.Because this upper limit is a possible strategy that satisfies $1 = F(y')$ we have:

$$K = \mu q I(py' - 1)\left[1 - q\right]^{N-1}$$

so that, using the assumption of a symmetric equilibrium, expected revenue in the mixed strategy equilibrium can be rewritten as $(py' - 1)\left[1 - q\right]^{N-1} = (pR - 1)\left[q(1 - F(R)) + 1 - q\right]^{N-1}$

From which $F(R)$ is obtained

$$F(R) = \frac{1}{q}\left\{1 - (1 - q)\left[\frac{py' - 1}{pR - 1}\right]^{\frac{1}{N-1}}\right\}$$

Denote by $\underline{R}$ the lower bound for R_i, which is defined by $F(\underline{R}) = 0$. Thus, $\underline{R}$ satisfies $1 = (1 - q)\left[\frac{py' - 1}{p\underline{R} - 1}\right]^{\frac{1}{N-1}}$ so that

$$p\underline{R} = 1 + (1 - q)^{N-1}(py' - 1)$$

9.2 Appendix B: Proof of proposition 9

The PDB problem under competition is:

$$\max_{S_C(p),P_F(p),q(p),p^*} \int_{p^*}^{1} \left\{\mu I\left[1-(1-q(p))^N\right][py-1-\lambda(S_C(p)+pP_F(p))] - NC(q(p))\right\} f(p)dp$$

$$\mu I(1-q)^{N-1}(py'+S_C-1)-C'(q(p))=0 \qquad (30)$$

$$p\,(y'+P_F(p))\geq 1$$

$$S_C(p)\geq 0; \qquad P_F(p)\geq 0; \qquad 1\geq q(p);$$

As before, denote by $\nu(p)$ and $\gamma(p)$ the Lagrangian multipliers respectively associated to constraint (30) and to the moral hazard constraint, and let $\delta(p)$ be the multiplier associated with $1\geq q(p)$.The Lagrangian conditions for, respectively, $S_C(p), P_F(p), q(p)$, and p^* become

$$-\lambda(1-(1-q(p))^N)f(p)+(1-q)^{N-1}\nu(p)\leq 0$$

$$-\lambda\mu I(1-(1-q(p))^N)f(p)+\gamma(p)\leq 0$$

$$\mu IN(1-q(p))^{N-1}(py-1-\lambda\left(S_C\left(p\right)+pP_F\left(p\right)\right))-NC'(q\left(p\right))-$$

$$-\frac{\nu(p)C''(q(p))+\delta(p)}{f(p)}=0$$

$$\mu I(1-(1-q(p^*))^N)(p^*y-1-\lambda(S_C(p^*)+pP_F(p^*)))-NC(q(p^*)) \quad = \quad 0$$

For firms with $p>\frac{1}{y'}$ it is optimal to set $P_F=0$. Because $q<1$, we have, for $S_C>0$ and, from the first order condition for P_F , we obtain $\nu(p)=\frac{\lambda(1-(1-q(p))^N)f(p)}{(1-q)^{N-1}}$. Using these ingredientes in the first order condition for q yields: $\mu I\left[(1-q(p))^{N-1}(py-1)-\lambda S_C\left(p\right)\right]-C'(q\left(p\right))-\lambda\frac{(1-(1-q(p))^N)}{N(1-q)^{N-1}}C''(q(p))=0$

Substracting (30) leads to

$$\mu I\left[(1-q(p))^{N-1}\left(p(y-y')-(1+\lambda)S_C\right)\right]-\lambda\frac{(1-(1-q(p))^N)}{N(1-q)^{N-1}}C''(q(p))=0$$

So that the optimal subsidy satisfies

$$S_C=\frac{1}{1+\lambda}\left\{p(y-y')-\frac{\lambda\frac{(1-(1-q(p))^N)}{N(1-q)^{N-1}}C''(q(p))}{\mu I(1-q(p))^{N-1}}\right\}$$

For firms with $p<\frac{1}{y'}$ a subsidy $P_F>0$ satisfying $p(y'+P_F)=1$ is enough for the firm to choose the good project. Substituting into the first order conditions, we obtain:

$$S_C=\frac{1}{1+\lambda}\left\{p(y-y')-\lambda(1-py')-\frac{\lambda\frac{(1-(1-q(p))^N)}{N(1-q(p))^{N-1}}C''(q(p))}{\mu I(1-q(p))^{N-1}}\right\}$$

Firms will receive subsidies, if any, when they satisfy both $py' < 1$ and $p\epsilon(p^*, \frac{1}{y'})$, but at an interest rate that implies a zero profit for the bank and $q(p) = 0$, so that the subsidy to the firm is ineffective if not accompanied by a credit subsidy.

9.3 Appendix C: Second best in case with collateral

Denote, as before, by $\nu(p)$ and $\gamma(p)$ the Lagrangian multipliers respectively associated to the first two constraints, and let $\delta(p)$ be the multiplier associated with $1 \geq q(p)$.

The first order conditions with respect to $S_C(p), P_F(p), q(p)$ and p^* are:

$$-\lambda q f(p) + \nu(p) \leq 0 \tag{31a}$$

$$-\lambda \mu q f(p) + \gamma(p)\Delta p \leq 0 \tag{31b}$$

$$\mu[(py-1)I^* - \left(p(y-1)\frac{V}{\nu_B} - (1-p)\delta V\right) - \lambda\left(S_C\left(p\right) + pP_F\left(p\right)\right)]I^*$$
$$-C'\left(q\left(p\right)\right) - \frac{\nu(p)C''(q(p)) + \delta(p)}{f(p)} = 0 \tag{31c}$$

$$\mu\left[q(p^*)(p^* y(p^*) - 1)I^* + (1-q(p))\left(p(y-1)\frac{V}{\nu_B} - (1-p)\delta V\right)\right]$$
$$-C(q(p^*)) - \lambda I(S_C(p^*) + pP_F(p^*)) = 0$$

The analysis of subsidies to screened firms is the same as before.The main impact of collateral will be on the optimal credit subsidy, as the option of collateralized lending decreases the benefits of screening.

Following the same procedure that we used to derive the optimal subsidy in the absence of collateral, we obtain, when a collateral V could be pledged with the bank:

$$S_C(p) = S_C^{cll}(p) \text{if } q(p) < 1$$
$$S_C\left(p\right) \leq S_C^{cll}(p) \text{ if } q(p) = 1$$

where $S_C^{cll}(p) = \{p(y-R(p)) - \left(p(y-R_V)\frac{1}{\nu_B} - (1-p)(1-\delta)\right)\frac{V}{I^*} - \frac{\lambda q}{\mu I^*}C''(q(p))\}\frac{1}{1+\lambda}$.

The introduction of collateral, however, also modifies the moral hazard problem for firms receiving collateralized loans, as they will now choose the positive net present value project taking into account the possible loss of collateral:

$$p(y - R_V\frac{V}{\nu_B} - (1-p)(1+\delta)V(\geq (p-\Delta p)(y - R_V\frac{V}{\nu_B} - (1+\Delta p - p)(1+\delta)V + B$$

This implies

$$R_V \leq y + (1+\delta)\nu_B - \frac{B\nu_B}{V\Delta p}$$

with, as intuition suggests, a much higher pledgeable cash flow due to the value of collateral to the firm

9.4 Appendix D: Unobservable Firms' Efforts

An alternative common form of moral hazard is the effort model, whereby firms choose the optimal level of effort (normalized to equal the probability of success) given its quadratic cost $C(e) = \frac{e^2}{2\beta}$. Under perfect observability and contractability of effort, a firm receiving a loan would make a repayment $I(1+\rho)$, so that the firm maximizes

$$\max_e epy - I(1+\rho) - \frac{e^2}{2\beta}$$

and the first best effort level $e^* = \beta py$ is obtained. Under moral hazard, the chosen level of effort $\widehat{e}$, for a repayment $R(p)$ will be the solution to:

$$\max_e ep\,(y - R(p)) - \frac{e^2}{2\beta}$$

so that $\widehat{e} = \beta p\,(y - R(p)) < \beta py$, where $\widehat{e}pR(p) = I(1+\rho)$

Consequently, a subsidy in conditional on success changes the objective function to $\max_e ep\,(y + P_F(p) - R(p)) - \frac{e^2}{2\beta}$ and its solution to

$$e = \beta p\,(y + P_F(p) - R(p)) \tag{32}$$

The equivalent second best problem will then have the added variable e and a different moral hazard constraint (32).

$$\max_{S_C(p),P_F(p),q(p),e,p^*} \int_{p^*}^{1} \left[\mu q(p)(epy-1)I - C(q(p)) - \lambda\mu q(p)I(S_C(p) + epP_F(p))\right] f(p)dp$$

$$\mu(epR(p) + S_C(p) - 1)I - C'(q(p)) = 0 \tag{33}$$

$$e = \beta p\,(y + P_F(p) - R(p))$$

$$S_C(p) \geq 0; \qquad P_F(p) \geq 0; \qquad 1 \geq q(p); \qquad e \leq 1$$

Denote by $\nu(p)$ and $\gamma(p)$ the Lagrangian multipliers respectively associated to constraints (33) and (32), and let $\delta(p)$ be the multiplier associated with $1 \geq q(p)$.

The first order conditions with respect to $S_C(p), P_F(p), q(p), e$ and p^* are:

$$-\lambda q f(p)+\nu(p) \quad \leq \quad (34)$$
$$-\lambda\mu qeIf(p)+\beta\gamma(p) \quad \leq \quad (35)$$
$$\mu I\left[epy-1-\lambda\left(S_C\left(p\right)+epP_F\left(p\right)\right)\right]-C'\left(q\left(p\right)\right)- -\frac{\nu(p)C''(q(p))+\delta(p)}{f(p)} \quad = \quad (36)$$
$$\mu q(p)I(py-\lambda pP_F(p))f(p) \quad +\nu(p)\mu IpR(p)-\gamma(p)= (37)$$
$$[\mu q(p^*)e(p^*y(p^*)-1)-C(q(p^*))-\lambda(S_C(p^*)+ep^*P_F(p^*))]=0 \tag{38}$$

Consider the case $S_C(p)>0; P_F(p)>0; 1>q(p); e<1$,

$$\begin{aligned}\nu(p) &= \lambda q f(p) \text{ and}\\ \beta p\gamma(p) &= \lambda\mu qeIf(p)\end{aligned}$$

Replacing in 36) and (37) yields, respectively :

$$\mu I\left[epy-\lambda\left(S_C\left(p\right)+epP_F\left(p\right)\right)\right]-C'\left(q\left(p\right)\right)-\lambda qC''(q(p))=0$$

and

$$\mu q(p)I(py-\lambda pP_F(p))f(p)+\lambda q(p)f(p)\mu IpR(p)-\frac{1}{\beta}\lambda\mu qeIf(p)=0$$

dividing by $\mu q(p)If(p)$ the expression simplifies to

$$py-\lambda pP_F(p)+\lambda pR(p)-\frac{1}{\beta}\lambda e=0 \tag{39}$$

$P_F(p)\geq 0$ if

$$py+\lambda pR(p)\geq\frac{1}{\beta}\lambda e \tag{40}$$

Expression (40), can simply be interpreted as the benefits of the subsidy being larger than its costs. As the benefits of the subsidy are derived from the incentive effect on e, we want the marginal cost of P_F, $\lambda\mu qIepf(p)$ to be lower than the benefits it generates. Now, the for each dollar increase of P_F, the impact on e is $\frac{de}{dP_F}=\beta p$. In turn, a unit increase in e will have an direct impact on the objective function of py, and an indirect impact in the incentives for the bank to increase its screening level, because a dollar of $epR(p)$ is equivalent to a dollar increase of $S_C(p)$. So, an increase in e leads to benefits of $py+\lambda pR(p)$, wich occur with probability $\mu q(p)If(p)$. So the net benefit condition for a subsidty is $\mu q(p)If(p)\beta p(py+\lambda pR(p))\geq\lambda\mu qIepf(p)$. Simplifying we obtain expression (40).

In order to obtain a condition for the positivity of P_F without the endogenous value of e, replacing e by its value $e=\beta p\left(y+P_F(p)-R(p)\right)$ in (39), we obtain:

$$y - \lambda P_F(p) + \lambda R(p) - \lambda\left(y + P_F(p) - R(p)\right) = 0$$

and

$$P_F = \frac{y(1-\lambda)}{2\lambda} + R(p)$$

So, for $\lambda < 1$, which seems a natural assumption, all firms will be subsidized: the impact on output and the reduction in the cost of subsidizing bank loans are sufficiently strong to yield this result.

Condition $e < 1$ implies, using (32) that $1 > \beta p\left(y + P_F(p) - R(p)\right).$

If instead, e reaches the corner solution, $e = 1$, and $\beta p\left(y + P_F(p) - R(p)\right) > 1$. This implies $\gamma(p) = 0$, and consequently $P_F(p) = 0$. Consequently, the firms that will receive a subsidy will be those for which $y - R(p) < \frac{1}{\beta p}$, which is the equivalent of $y - R(p) < \frac{B}{\Delta p I}$ in our modeling approach.

Because the moral hazard problem has changed, the firms to which the subsidy will be granted has also changed. While, in the presence of the private benefits switch to private benefits the subsidy was to those firms that had insufficient rents to provide the right incentives (but were close enough), now the subsidy will go to any firm with an effort level lower than $e = 1$.